Calculus

أمثلة و تمارين

لطلبة كليات الهندسة

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Old Exam 5.1 :

(092)

2. Using four rectangles and right endpoints, the area under the graph of

$$f(x) = \sin x$$

from $x = 0$ to $x = \pi$ is approximately equal to

(a) $\dfrac{\pi(1+\sqrt{2})}{4}$

(b) $\dfrac{\sqrt{2}(1+\pi)}{4}$

(c) $\dfrac{\pi}{4}$

(d) $\dfrac{\pi(1-\sqrt{2})}{2}$

(e) $\pi(1+\sqrt{2})$

(091)

1. Using four rectangles and left endpoints, the area under the graph of $f(x) = x^2 - 2x$ from $x = 2$ to $x = 6$ is approximately equal to

(a) 26

(b) 23

(c) 35

(d) 38

(e) 40

(083)

1. Using four rectangles and midpoints, the area under the graph of $f(x) = x^2 + 2x$ from $x = 0$ to $x = 8$ is approximately equal

(a) 116

(b) 232

(c) 102

(d) 223

(e) 320

(082)

1. Using four rectangles and right end points, the estimated area under the graph of $f(x) = 1 + \dfrac{x^2}{4}$ from $x = -2$ to $x = 6$ is

(a) 36

(b) 40

(c) 18

(d) 24

(e) 34

(081)

5. Using three approximating rectangles and midpoints, the area under the graph of $f(x) = \dfrac{x}{x-1}$ from $x = 2$ to $x = 8$ is approximately equal to

(a) $\dfrac{29}{3}$

(b) $\dfrac{41}{12}$

(c) $\dfrac{47}{6}$

(d) $\dfrac{59}{6}$

(e) $\dfrac{43}{6}$

(073)

5. The limit $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\frac{\pi}{4n}\tan\left(\frac{\pi i}{4n}+\frac{\pi}{3}\right)$ can be interpreted as the area under the graph of the function

(a) $y=\tan\left(x+\frac{\pi}{3}\right),\quad 0\le x\le\frac{\pi}{4}$

(b) $y=\tan\frac{1}{4}\left(x+\frac{\pi}{3}\right),\quad 0\le x\le\frac{\pi}{4}$

(c) $y=\tan\left(\frac{x}{4}+\frac{\pi}{3}\right),\quad 0\le x\le\frac{\pi}{4}$

(d) $y=\frac{\pi}{3}+\tan x,\quad 0\le x\le\frac{\pi}{4}$

(e) $y=\tan\left(x+\frac{\pi}{3}\right),\quad \frac{\pi}{4}\le x\le\frac{\pi}{3}$

6. An expression for the area under the graph of $f(x)=4x-x^2,\ 2\le x\le 4,$ as a limit and using right endpoints is

(a) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(\frac{8}{n}-\frac{8}{n^3}i^2\right)$

(b) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(\frac{8}{n}-\frac{4}{n}i-\frac{8}{n^3}i^2\right)$

(c) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(\frac{12}{n}-\frac{16}{n^3}i^2\right)$

(d) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(\frac{4}{n}-\frac{8}{n^3}-\frac{16}{n^3}i^2\right)$

(e) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(\frac{8}{n}+\frac{16}{n^3}i^2\right)$

(072)

10. The limit $\lim_{n \to \infty} \sum_{i=1}^{n} \left[\frac{4i}{n^2} + \frac{6}{n} \right]$ can be interpreted as

(a) area under the graph of $y = x$ on $[0, 2]$

(b) area under the graph of $y = x + 3$ on $[3, 5]$

(c) area under the graph of $y = x$ on $[3, 5]$

(d) area under the graph of $y = 4x^2 + 6x$ on $[3, 5]$

(e) area under the graph of $y = 2x + 3$ on $[1, 3]$

(071)

1. An estimate of the area under the graph of $f(x) = 16 - x^2$ from $x = 0$ to $x = 4$ using four approximating rectangles and left endpoints is

(a) 50

(b) 40

(c) 30

(d) 20

(e) 45

(062)

1. The **estimated area** under the graph of $f(x) = 20 - 2x^2$ from $x = -2$ to $x = 3$ using **five approximating rectangles** and **right end-points** is

(a) 70

(b) 80

(c) 90

(d) 75

(e) 60

(061)

7. An estimate of the area under the graph of $y = \frac{1}{x}$ from $x = 2$ to $x = 6$ using four approximating rectangles and right endpoints is

(a) $31/20$

(b) $19/20$

(c) $29/20$

(d) $21/20$

(e) $17/20$

Answer Key :

Question	Answer
2 (092)	A
1 (091)	A
1 (083)	B
1 (082)	A
5 (081)	C
5 (073)	A
6 (073)	A
10 (072)	C
1 (071)	A

1 (062)	A
7 (061)	B

Old Exam 5.2 :

(092)

8. $\int_{-3}^{0} \left(|\, x - 1| + \sqrt{9 - x^2} \right) dx =$
(Hint: You may interpret the integral as an area)

(a) $\dfrac{9\pi + 30}{4}$

(b) $\dfrac{9\pi + 26}{4}$

(c) $\dfrac{9\pi + 34}{4}$

(d) $\dfrac{7\pi + 30}{4}$

(e) $\dfrac{7\pi + 34}{4}$

9. $\lim_{n \to \infty} \dfrac{2}{n} \sum_{i=1}^{n} \dfrac{1}{1 + \left(\frac{i-1}{n}\right)^2} =$

(a) $\dfrac{\pi}{2}$

(b) $\dfrac{\pi}{4}$

(c) 0

(d) 1

15. The region whose area is equal to $\lim_{n\to\infty} \frac{4}{3}\left(\frac{\pi+3}{n}\right)\sum_{i=1}^{n} \sin\left(\frac{\pi\, i+3i-3n}{3n}\right)^2$ is the region

(a) under the graph of $y = 4\sin x^2$ from -1 to $\frac{\pi}{3}$.

(b) under the graph of $y = \sin x^2$ from 1 to $\frac{\pi}{3}$.

(c) under the graph of $y = \sin\left(\frac{x^2}{4}\right)$ from -1 to $\frac{\pi}{4}$.

(d) under the graph of $y = 4\sin x^2$ from 1 to $\frac{\pi}{4}$.

(e) under the graph of $y = \frac{4}{3}\sin x^2$ from 3 to π.

20. Let m and M be the absolute minimum and the absolute maximum values respectively, of an integrable function f over a closed interval $[3, 5]$. If an estimation, based on m and M, of the integral $\int_3^5 f(x)dx$ lies in the interval $[A, B]$, then $A+B=$

(a) $2(M+m)$

(b) $2(M-m)$

(c) $8(M+m)$

(d) $8(M-m)$

(e) $2Mm$

(091)

8. $\int_6^4 f(x)dx + \int_4^{-1} f(x)dx - \int_6^{-3} f(x)dx =$

(a) $\int_{-3}^{-1} f(x)dx$

(b) $\int_{-1}^{-3} f(x)dx$

(c) $\int_4^{-3} f(x)dx$

(d) $\int_{-1}^{6} f(x)dx$

(e) $\int_4^{-1} f(x)dx$

11. If R_n is the Riemann sum for

$$f(x) = 3 + \frac{2}{9}x^2, \quad 0 \leq x \leq 3,$$

with n subintervals and taking sample points to be the right endpoints, then $R_n =$

(a) $9 + \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$

(b) $3 + \frac{2}{9}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$

(c) $9 + \frac{1}{27}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$

(d) $1 + \frac{2}{3}\left(1+\frac{2}{n}\right)\left(2+\frac{1}{3n}\right)$

(e) $3 + \frac{1}{6}\left(1-\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$

20. $$\lim_{n\to+\infty}\sum_{i=1}^{n}\left[2+\left(1+\frac{4i}{n}\right)^7\right]\frac{5}{n}=$$

(a) $\displaystyle\int_0^5\left[2+\left(1+\frac{4}{5}x\right)^7\right]dx$

(b) $\displaystyle\int_0^5\left[2+(1+4x)^7\right]dx$

(c) $\displaystyle\int_2^7\left[2+(1+4x)^7\right]dx$

(d) $\displaystyle\int_2^7\left[2+\left(1+\frac{4}{5}x\right)^7\right]dx$

(e) $\displaystyle\int_0^5\left(1+\frac{4}{5}x\right)^7dx$

(083)

5. $\int_{-5}^{0} \left(2x - 4\sqrt{25 - x^2}\right) dx =$

(a) 25π

(b) $-25(1 + \pi)$

(c) $-25\left(1 + \frac{\pi}{4}\right)$

(d) $25(1 - \pi)$

(e) $25 - \frac{\pi}{4}$

7. $\sum_{i=1}^{n} \left(5 - \frac{4i}{n}\right) =$

(a) $2n^2 + 3n$

(b) $3 - 2n$

(c) $3n - 2n^2$

(d) $2n^2 + 4n + 1$

(e) $3n - 2$

7. Using the definition of the Area and definite integral, the value of the limit $\lim_{n\to\infty}\sum_{i=1}^{n} e^{4+\frac{3i}{n}} \cdot \frac{2}{n}$ is [Hint: Express the limit as a definite integral].

(a) $\frac{2}{3}[e^7 - e^4]$

(b) e^4

(c) $\frac{2}{3}e^{7/4}$

(d) does not exist

(e) $e^{-3} + e^{-4}$

8. If $f(x) = \begin{cases} x & 0 \le x \le 1 \\ \sqrt{1-(x-1)^2} & 1 \le x \le 2 \end{cases}$, and definite ingeral is interpreted as an area, then the value of the integral $\int_0^2 f(x)dx$ is

(a) $(2+\pi)/4$

(b) $(1+\pi)/4$

(c) $(3+\pi)/4$

(d) $(4+\pi)/4$

(e) $(5+\pi)/4$

19. If we use the definition of $\ln x$ as a definite integral, then an approximation of $\ln 2$ using two rectangles and the sample points to be midpoints is equal to

(a) $\frac{24}{35}$

(b) $\frac{13}{15}$

(c) $\frac{12}{35}$

(d) $\frac{11}{15}$

(e) $\frac{29}{35}$

20. The limit $\lim_{n \to +\infty} \sum_{i=1}^{n} \frac{1}{i+n}$ can be interpreted as the

(a) area under the graph of the function $y = \frac{1}{x}$ on $[2, 4]$

(b) area under the graph of the function $y = \frac{1}{x}$ on $[0, 3]$

(c) area under the graph of the function $y = \ln x$ on $[2, 4]$

(d) area under the graph of the function $y = \ln x$ on $[1, 3]$

(e) area under the graph of the function $y = x$ on $[2, 4]$

8. Using the definition of the definite integral, the value of the limit

$$\lim_{n\to+\infty} \sum_{i=1}^{n} \frac{2}{n}\sqrt{4+\frac{3i}{n}}$$

is equal to

(a) $\dfrac{32}{\sqrt{7}}$

(b) $\dfrac{4}{\sqrt{3}}$

(c) $\dfrac{2}{3}(7\sqrt{7}-8)$

(d) $\dfrac{4}{9}(7\sqrt{7}-8)$

(e) $\dfrac{28\sqrt{7}}{9}$

20. If $\int_{-1}^{2} f(x)\,dx = 4$ and $\int_{1}^{2} f(2x)\,dx = 1$, then $\int_{-1/3}^{4/3} f(3x)\,dx =$

(a) $\dfrac{2}{3}$

(b) 3

(c) 4

(d) 5

(e) 2

(073)

3. The Riemann sum for $f(x) = \dfrac{15}{x}$, $1 \le x \le 3$ with four subintervals, taking the sample points to be right endpoints, is equal to

(a) $\dfrac{57}{4}$

(b) $\dfrac{29}{4}$

(c) $\dfrac{63}{4}$

(d) $\dfrac{59}{8}$

(e) $\dfrac{63}{8}$

7. $\displaystyle \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n} + 2\right)^2 \frac{1}{n} =$

(a) $\dfrac{19}{3}$

(b) $\dfrac{29}{6}$

(c) $\dfrac{31}{3}$

(d) $\dfrac{19}{6}$

(e) $\dfrac{38}{3}$

2. $$\lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^3\right)=$$

(072)

(a) $\frac{1}{2}\int_1^3 (1+x)^3\,dx$

(b) $\int_1^3 x^3\,dx$

(c) $\int_1^3 (1+x)^3\,dx$

(d) $\frac{1}{2}\int_1^3 x^3\,dx$

(e) $\int_0^3 x^3\,dx$

3. The Riemann sum for $f(x) = \sin x$, $0 \le x \le \pi$, with 6 equal subintervals, taking the sample points to be left endpoints, is equal to

(a) $\dfrac{(2-\sqrt{3})\pi}{6}$

(b) $\dfrac{(2+\sqrt{3})\pi}{6}$

(c) $\dfrac{(2+\sqrt{2})\pi}{6}$

(d) $\dfrac{\pi}{6}$

(e) $\dfrac{(3+\sqrt{2})\pi}{6}$

5. Using area under curves to evaluate the integral

$$\int_{-2}^{2}(|x| + \sqrt{4 - x^2})\ dx,$$

we get

(a) $8 + \pi$

(b) $4 + 2\pi$

(c) $4 + 4\pi$

(d) $4 + \pi$

(e) 2π

9. If $A = \int_0^{\pi/2} \sqrt{1 + \sin x}\ dx$, then

(a) $\frac{\pi}{2} \leq A \leq \frac{\pi}{\sqrt{2}}$

(b) $\frac{\pi}{\sqrt{2}} \leq A \leq \pi$

(c) $1 + \frac{\pi}{2} \leq A$

(d) $A \leq 1$

(e) $1 \leq A \leq \frac{\pi}{2}$

(071)

2. The value of the limit $\lim_{n \to +\infty} \sum_{i=1}^{n} \left(\frac{4i}{n^2} + \frac{3}{n} \right)$ is equal to

(a) 5

(b) 7

(c) 1

(d) -4

(e) -8

6. By interpreting the integral $\int_{-2}^{2} (3 + \sqrt{4 - x^2})\, dx$ in terms of areas, its value is equal to

(a) $12 + 2\pi$

(b) $6 + 2\pi$

(c) $6 + \pi$

(d) $12 + \pi$

(e) $6 + 4\pi$

13. When expressing the limit

$$\lim_{n\to+\infty} \sum_{i=1}^{n} \frac{\pi}{3n} \cot\left(\frac{\pi}{6} + \frac{i\pi}{3n}\right)$$

as a definite integral, it becomes

(a) $\displaystyle\int_{\pi/6}^{\pi/2} \cot x \, dx$

(b) $\displaystyle\int_{\pi/6}^{\pi/3} \cot x \, dx$

(c) $\displaystyle\int_{\pi/6}^{\pi/2} \cot\left(\frac{\pi}{6} + x\right) dx$

(d) $\displaystyle\int_{\pi/6}^{\pi/4} x \cot\left(\frac{\pi}{6} + x\right) dx$

(e) $\displaystyle\int_{\pi/3}^{\pi/2} \cot x \, dx$

18. $\lim_{x \to 0} \left(1 + \frac{1}{2}x\right)^{\frac{1}{2x}} =$

(a) $\sqrt[4]{e}$

(b) $\sqrt{e}$

(c) e

(d) e^2

(e) e^4

(063)

2. $\int_{-2}^{0} (2 + \sqrt{4 - x^2}) dx =$

(a) $\pi + 1$
(b) $\pi + 4$
(c) π
(d) $\frac{\pi}{2}$
(e) $\pi - 2$

3. Let $I = \int_{-2}^{3} xe^{-x}dx$. Then, we can say

(a) $-10e^2 \leq I \leq \frac{5}{e}$

(b) $-10e^2 \leq I \leq -8e^2$

(c) $\frac{3}{e^3} \leq I \leq \frac{1}{e}$

(d) $0 \leq I \leq 1$

(e) $I \geq \frac{5}{e}$

8. $\lim\limits_{n \mapsto \infty} (\frac{1}{n^3} \sum\limits_{i=1}^{n} (i^2) =$

(a) $\int\limits_0^2 x^2 dx$

(b) $\int\limits_0^2 \frac{x^2}{8} dx$

(c) $\int\limits_0^1 2x^2 dx$

(d) $\int\limits_0^2 x dx$

(e) $\int\limits_0^2 2x^3 dx$

(062)

4. If $\int_{-1}^{6} f(x)dx = 12$, $\int_{-1}^{4} f(x)dx = 16$ and $\int_{5}^{6} f(x)dx = -18$, then $\int_{4}^{5} f(x)dx$ is equal to

(a) 14

(b) -10

(c) 22

(d) 10

(e) 28

9. The value of the integral $\int_{0}^{2}(4+\sqrt{4-x^2})dx$ by interpreting it in terms of areas is

(a) $8+\pi$

(b) $8+\dfrac{\pi}{4}$

(c) $4+2\pi$

(d) $4+\dfrac{\pi}{4}$

(e) $6+2\pi$

15. The limit $\lim_{t \to 0} (1 + 2t)^{3/t}$ is equal to

(a) e^6

(b) $e^{3/2}$

(c) 6

(d) $\dfrac{3}{2}$

(e) e^2

(061)

2. Which one of the following expressions approximates better the area under the curve of $y = x^2 \sin x$ from $x = 0$ to $x = \pi$?

(a) $\displaystyle\sum_{k=1}^{n} \frac{\pi^2}{2n^2} \left(\frac{k^2\pi}{2n} \sin \frac{k\pi}{2n} \right)$

(b) $\displaystyle\sum_{k=1}^{n-1} \frac{\pi^2}{n^2} \left(\frac{k^2\pi}{n} \sin \frac{k\pi}{\pi} \right)$

(c) $\displaystyle\sum_{k=1}^{n} \frac{\pi}{n} \left(\frac{2k-1}{4} \right) \sin \frac{(2k-1)\pi}{4n}$

(d) $\displaystyle\sum_{k=1}^{n-1} \frac{k^3\pi^2}{n^3} \sin \frac{k^2\pi}{n}$

(e) $\displaystyle\sum_{k=1}^{n} \frac{\pi}{n} \left(\frac{k^2\pi^2}{n^2} \sin \frac{(k-1)\pi}{n} \right)$

11. Which one of the following relations is **correct**?

(a) $\int_0^{\pi/4} \sin^3 x \, dx \geq \int_0^{\pi/4} \sin^2 x \, dx$

(b) $\int_{-\pi/2}^{\pi/2} \sin x \, dx > 0$

(c) $\frac{\pi}{6} \leq \int_{\pi/6}^{\pi/2} \sin x \, dx \leq \frac{\pi}{3}$

(d) $\int_0^{\pi/2} \sin x \, dx < \int_0^{\pi/2} \cos x \, dx$

(e) $\frac{\pi}{3} \leq \int_{\pi/6}^{\pi/2} \sin x \, dx \leq \frac{\pi}{2}$

(061)

14. Evaluating the integral $\int_0^3 \sqrt{9 - x^2} dx$ by interpreting it in terms of areas, we get

(a) $81\pi^2/4$

(b) $9\pi/4$

(c) $9\pi/2$

(d) $3\pi^2/2$

(e) $81\pi/2$

Answer Key :

Question	Answer
8 (092)	A
9 (092)	A
20 (092)	A
15 (092)	A
8 (091)	A
11 (091)	A
20 (091)	A
5 (083)	B
7 (083)	E
7 (082)	A
8 (082)	A
19 (082)	A
20 (082)	A
8 (081)	D
20 (081)	E
3 (073)	A
7 (073)	A
2 (072)	D
3 (072)	B
5 (072)	B
9 (072)	A
2 (071)	A

6 (071)	A
13 (071)	A
18 (071)	A
2 (063)	--
3 (063)	--
8 (063)	--
4 (062)	A
9 (062)	A
15 (062)	A
2 (061)	B
11 (061)	C
14 (061)	B

Old Exam 5.3 :

(092)

3. $\displaystyle\int_0^{\frac{1}{2}} \left(\frac{6}{\sqrt{1-t^2}} + \frac{12t-2}{3\sqrt{t}} \right) dt =$

(a) π

(b) $\pi + \sqrt{2}$

(c) $\pi + 2\sqrt{2}$

(d) $\pi + 3\sqrt{2}$

(e) $\pi + 4\sqrt{2}$

5. If $F(x) = \int_1^x f(t)\,dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\,du$, then $F''(2) =$

(a) $\sqrt{257}$

(b) $\sqrt{255}$

(c) $\sqrt{253}$

(d) $\sqrt{259}$

(e) $\sqrt{261}$

16. The slope of the line tangent to the curve $g(x) = \int_0^{x^3} \sqrt{t + e^t}\,dt$ at $x = 2$ is

(a) $12\sqrt{8+e^8}$

(b) $8\sqrt{8+e^8}$

(c) $8\sqrt{2+e^2}$

(d) $12\sqrt{2+e^2}$

(e) $12\sqrt{8+e^2}$

(091)

4. $\int_0^{2\sqrt{2}} (3 - 2\sqrt{8 - x^2})\, dx =$

(a) $6\sqrt{2} - 4\pi$

(b) $6\sqrt{2} - 2\pi$

(c) $6\sqrt{2} - 8\pi$

(d) $3\sqrt{2} - 2\pi$

(e) $2\sqrt{2}$

7. If $G(x) = \int_{\sin x}^{\cos(3x)} \frac{1}{\sqrt{1 + 4t^2}}\, dt$, then $G'\left(\frac{\pi}{2}\right) =$

(a) 3

(b) $\frac{16}{5}$

(c) $\frac{-14}{5}$

(d) $\frac{3}{5}$

(e) 2

9. $\int_1^4 \frac{d}{dx}\left(\frac{\ln x}{\sqrt{x}}\right) dx =$

(a) $\ln 2$

(b) $-1 + \ln 2$

(c) $\ln 4$

(d) $2 + \ln 4$

(e) cannot be evaluated

19. If $\int_0^1 f(3x-5)dx = 4$, then $\int_{-5}^{-2} f(x)dx =$

(a) 12

(b) 4

(c) 3

(d) $\frac{1}{4}$

(e) $\frac{4}{3}$

(083)

4. $\dfrac{d}{dx}\left[\int_{\sqrt{x}}^{2} \cos\left(t^2\right) dt\right] =$

(a) $\dfrac{\cos x}{\sqrt{x}}$

(b) $\cos 4 - \cos x$

(c) $\sin 4 - \sin x$

(d) $\dfrac{\sin 4}{4} - \dfrac{\sin x}{2\sqrt{x}}$

(e) $-\dfrac{\cos x}{2\sqrt{x}}$

8. $\lim\limits_{n\to+\infty} \frac{1}{n}\left(\sqrt[3]{\frac{1}{n}}+\sqrt[3]{\frac{2}{n}}+\sqrt[3]{\frac{3}{n}}+\ldots+\sqrt[3]{\frac{n}{n}}\right)=$

[Hint: First express the limit as a definite integral]

(a) $\frac{3}{4}$

(b) 0

(c) $\sqrt[3]{4}$

(d) 1

(e) $\frac{3}{2}$

9. Which one of the following statements is **FALSE**: (f is continuous on [a,b])

(a) If $f(x) \leq 0$ on [a,b], then $\int_a^b f(x)dx \leq 0$.

(b) $\int_a^b 4\ f(x)dx = 4\int_a^b f(x)dx$.

(c) If $\int_a^b f(x)dx = 7$, then $\int_a^b f(t)dt = 7$.

(d) If $\int_a^b f(x)dx = 0$, then $f(x) = 0$ for all x in $[a, b]$

(e) $\int_a^b f(x)dx + \int_b^a f(x)dx = 0$.

14. If $15 + \int_3^x e^{-t} f(t)\, dt = 5x$ for all x, then $f(0) + f'(0) =$

(a) 15e

(b) 5e

(c) 3

(d) 10

(e) 5

15. If f is continuous on $[0, 1]$ and $\int_0^1 f(x)\,dx = 2$, then $\int_0^1 f(1-x)\,dx =$

(a) -2

(b) 1

(c) 0

(d) -1

(e) 2

(082)

5. If $F(x) = \int_{\frac{1}{2}}^{x} f(t)\ dt$ and $f(t) = \int_{\frac{1}{2}}^{t^2} \frac{\sqrt{1+u^2}}{u}\ du$, then $F''(1) =$

(a) $2\sqrt{2}$

(b) $\sqrt{2}$

(c) $\frac{\sqrt{2}}{2}$

(d) $3\sqrt{2}$

(e) $\frac{\sqrt{2}}{6}$

6. If $y = \int_{\sqrt{x}}^{x^3} \sqrt{t} \sin t\ dt$, then $\frac{dy}{dx} =$

(a) $3\sqrt{x^7}\sin(x^3) - \frac{1}{2\sqrt[4]{x}}\sin\sqrt{x}$

(b) $\sqrt{x^3}\sin x^3 - \sqrt[4]{x}\sin\sqrt{x}$

(c) $\sqrt{x}\sin x^3 - \frac{1}{\sqrt[4]{x}}\sin\sqrt{x}$

(d) $\sqrt{x^3}\sin\sqrt{x} - x^2\sin x^3$

(e) $x^3\sin\sqrt{x} - \sqrt{x}\sin x^3$

12. If $\int_0^1 f(x)dx = \pi$, then $\int_0^{\pi/4} f(\sin 2x)\cos 2x \; dx$ is

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) π

(d) 2π

(e) 4π

(081)

2. If $f(x) = \begin{cases} \dfrac{3}{x} & \text{if } x \leq -1 \\ -3 & \text{if } x > -1, \end{cases}$ then $\int_{-3}^{0} f(x)\,dx$

(a) is equal to $3 - 3\ln 3$

(b) does not exist

(c) is equal to $3 + 3\ln 3$

(d) is equal to $-3 - \ln 3$

(e) is equal to $-3 - 3\ln 3$

3. If $g(x) = \int_{e^x}^{1} t \ln t \, dt$, then $g'(x) =$

(a) e^{2x}

(b) xe^{x}

(c) $-e^{x}$

(d) $-xe^{x}$

(e) $-xe^{2x}$

9. If $F(x) = \int_x^{x^2} \frac{\sin(2t)}{t^2} dt$, then $F(1) + F'(1) =$

(a) 0

(b) $\frac{\sin 2}{2}$

(c) $\sin 2$

(d) $1 + \sin 2$

(e) $3 \sin 2$

(073)

2. If $f(x) = \begin{cases} 3^x & 0 \le x \le 1 \\ 3x^2 + 1 & 1 < x \le 2 \end{cases}$, then $\int_0^2 f(x)dx =$

(a) $8 + \frac{2}{\ln 3}$

(b) $8 + \frac{3}{\ln 3}$

(c) $10 + \frac{1}{\ln 3}$

(d) $8 - \frac{2}{\ln 3}$

(e) $6 + \frac{4}{\ln 3}$

9. Which one of the following integrals **exists** according to the Fundamental Theorem of Calculus?

(a) $\displaystyle\int_0^{\pi/4} \cot\left(x+\frac{\pi}{2}\right)\, dx$

(b) $\displaystyle\int_0^{1} \ln x\, dx$

(c) $\displaystyle\int_{\pi/4}^{\pi} \sec x\, dx$

(d) $\displaystyle\int_{-5}^{5} \frac{2}{x^7}\, dx$

(e) $\displaystyle\int_0^{2} \frac{e^{x-1}}{(x-1)^2}\, dx$

12. Which one of the following statements is **FALSE** about the function $f(x)=\displaystyle\int_1^x \frac{1}{t}\, dt,\quad x>0$?

(a) $f(x_1+x_2)=f(x_1)+f(x_2)$ for all $x_1, x_2 > 0$

(b) f is increasing for all $x>0$

(c) $\displaystyle\lim_{x\to 0^+} f(x) = -\infty$

(d) $e^{f(e)}=e$

(e) The graph of f is concave downward for all $x>0$.

(072)

4. If f is a continuous function such that

$$\int_1^x e^{-t} f(t)\, dt = 3 + x \sin x$$

for all x, then $f(x) =$

(a) $e^x + \sin x$

(b) $x \cos x$

(c) $e^x(x \cos x + \sin x)$

(d) $xe^{-x} \sin x$

(e) $e^{-x} \cos x$

7. If $F(x) = \int_{x^2}^{x^3} \sqrt{1+t^2}\, dt$, then $F'(\sqrt{2}) =$

(a) $18 - 2\sqrt{10}$

(b) $18 + 2\sqrt{10}$

(c) 18

(d) $12\sqrt{2} - 2\sqrt{5}$

(e) $2\sqrt{10} - 18$

(071)

12. If f' is continuous on $[1,3]$, then $\int_1^3 f'(x)dx =$

(a) $f(3) - f(1)$

(b) $f'(3) - f'(1)$

(c) $f(2)$

(d) $f(1) - f(3)$

(e) $f(3) + f(1)$

14. If f is continuous and $\int_3^5 f(x)\,dx = 8$, then $\int_0^1 f(2x+3)\,dx =$

(a) 4

(b) 5

(c) 6

(d) 7

(e) 8

17. If $f(x) = \int_1^{x^3+3x} (t^3+1)^{20}\,dt$, then $f'(0)$ equals

(a) 3

(b) 0

(c) $\frac{1}{7}$

(d) $\frac{3}{20}$

(e) $\frac{20}{3}$

4. $\frac{d}{dx}\int_{x^2}^{x^3}(t^2+1)dx =$

(a) $x^6 - x^2$

(b) $x^3 - x^2$

(c) 0

(d) $3x^2 - 2x$

(e) $3x^8 - 2x^5 + 3x^2 - 2x$

(063)

5. If $\int_1^3 f(x)dx = \int_3^5 -2f(x)dx = \int_1^3(g(x)+2)dx = \int_3^5(g(x)+x)dx = 3$, then $\int_1^5(f(x)-g(x))dx =$

(a) 6

(b) $-\frac{9}{2}$

(c) -9

(d) $\frac{15}{2}$

(e) $-\frac{1}{2}$

(062)

13. Let $y = \int_{\sin x}^{x^2} \tan(u^3)du$. Then $\dfrac{dy}{dx}$ is

(a) $2x\tan(x^6) - \cos x \tan(\sin^3 x)$

(b) $\sec^2(x^2) - \sec^2(\sin^3 x)$

(c) $\tan(x^3)(x^2 - \sin x)$

(d) $\tan(x^6) - \tan(\sin^3 x)$

(e) $\sec(x^2)\tan(x^2) - \sec(\sin^3 x)\tan(\sin^3 x)$

20. If $G(u) = \int_1^u g(x)dx$ where $g(x) = \int_1^{x^2} \frac{\sqrt{9+t^2}}{t}\, dt$, then $G''(2)$ is equal to

(a) 5

(b) $\frac{\sqrt{\pi}}{2} - \sqrt{10}$

(c) $\frac{\sqrt{\pi}}{2}$

(d) $\sqrt{\pi}$

(e) 25

(061)

6. Which one of the following properties is **false**?

(a) $e = \lim_{x \to +\infty} (1+x)^{1/x}$

(b) $\ln\left(\frac{s}{t}\right) = -\ln t + \ln s$

(c) $(\ln a)\frac{d}{dt}(\log_a t) = \frac{1}{t}$

(d) $\frac{d}{dt}(3^{t-1}) = 3^{t-1} \ln 3$

(e) $u^{n-m} = \frac{u^n}{u^m}$

9. Using the Fundamental Theorem of Calculus, we find that the derivative of the function $f(x) = \int_1^{x^2} t \sin t \, dt$ is equal to

(a) $2x^3 \sin x^2$

(b) $x^2 \sin x^2 - \sin 1$

(c) $x^2 \sin x^2$

(d) $x \sin x$

(e) $x^3 \sin x^2$

Answer Key :

Question	Answer
3 (092)	A
5 (092)	A
16 (092)	A
4 (091)	A
7 (091)	A
9 (091)	A
19 (091)	A
4 (083)	E
8 (083)	A
9 (083)	D
14 (083)	D
15 (083)	E
5 (082)	A
6 (082)	A
12 (082)	A
2 (081)	E
3 (081)	E
9 (081)	C
2 (073)	A
9 (073)	A
12 (073)	A
4 (072)	C

7 (072)	A
12 (071)	A
14 (071)	A
17 (071)	A
4 (063)	--
5 (063)	--
13 (062)	A
20 (062)	A
6 (061)	A
9 (061)	A

Old Exam 5.4 :

(092)

4. $\displaystyle\int \frac{(x-2)^3}{x^2}\,dx =$

(a) $\dfrac{x^2}{2} - 6x + 12\ln|x| + \dfrac{8}{x} + c$

(b) $\dfrac{x^2}{2} + 6x + 12\ln|x| - \dfrac{8}{x} + c$

(c) $\dfrac{x^2}{2} - 6x + 12\ln|x| - \dfrac{8}{x} + c$

(d) $\dfrac{x^2}{2} - 6x + 6\ln|x| - \dfrac{4}{x} + c$

(e) $\dfrac{x^2}{2} + 6x - 12\ln|x| + \dfrac{8}{x} + c$

10. $\int \sin^2 x \, dx =$

(a) $\dfrac{x}{2} - \dfrac{\sin\ 2x}{4} + c$

(b) $-\cos\ x + c$

(c) $\dfrac{1}{2}\cos^2 x + c$

(d) $\dfrac{1}{2}\cos\ 2x + c$

(e) $\dfrac{x}{2} - \cos\ x + c$

14. A particle moves along a line so that its velocity at time t is $v(t) = t - t^2$. The distance traveled by the particle during the time period $0 \leq t \leq 2$ is:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

(091)

2. $\int (\sqrt[4]{y} + y)^2 \, dy =$

(a) $\frac{2}{3}y^{3/2} + \frac{8}{9}y^{9/4} + \frac{1}{3}y^3 + C$

(b) $\frac{2}{3}y^{3/2} + \frac{4}{9}y^{9/4} + \frac{1}{3}y^3 + C$

(c) $\frac{4}{5}y^{5/4} + \frac{2}{9}y^{9/4} + \frac{1}{3}y^3 + C$

(d) $\frac{1}{2}y^2 + \frac{1}{3}y^3 + C$

(e) $\frac{(\sqrt[4]{y} + y)^3}{3} + C$

10. $\int (\tan^2 t - \cot^2 t) dt =$

(a) $\tan t + \cot t + C$

(b) $\sec t + \csc t + C$

(c) $\dfrac{1}{3} \tan^2 t - \dfrac{1}{3} \cot^3 t + C$

(d) $t + C$

(e) $t + \tan t + \sec t + C$

13. If the velocity of a particle moving in a straight line is given by

$$v(t) = \frac{1}{2} - \cos t, \quad t \geq 0$$

then the distance traveled during the time inteval $\left[0, \frac{\pi}{2}\right]$ is

(a) $\sqrt{3} - 1 - \frac{\pi}{12}$

(b) $\frac{\pi}{4} - 1$

(c) $\sqrt{3} - 1 + \frac{\pi}{12}$

(d) $2 - \frac{\pi}{6}$

(e) $\sqrt{3} + 1 + \frac{\pi}{12}$

15. $$\int x\sqrt{2x-1}\,dx =$$

(a) $\frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C$

(b) $\frac{2}{5}(2x-1)^{5/2} + \frac{1}{3}(2x-1)^{3/2} + C$

(c) $\frac{2}{3}(2x-1)^{3/2} + 2(2x-1)^{1/2} + C$

(d) $\frac{\sqrt{2x-1}}{x} + C$

(e) $\frac{1}{10}\left(\frac{x+1}{2}\right)^{5/2} + \frac{2}{3}\left(\frac{x+1}{2}\right)^{1/2} + C$

(083)

2. $\int \frac{(x+1)^2}{\sqrt[3]{x}}\,dx =$

(a) $\frac{3}{8}x^{\frac{8}{3}} + \frac{6}{5}x^{\frac{5}{3}} + \frac{3}{2}x^{\frac{2}{3}} + C$

(b) $\frac{3}{8}x^{\frac{8}{3}} + \frac{3}{5}x^{\frac{5}{3}} + \frac{3}{2}x^{\frac{2}{3}} + C$

(c) $\frac{1}{2}(x+1)^3 \cdot x^{\frac{2}{3}} + C$

(d) $\frac{3}{5}x^{\frac{5}{3}} + \frac{3}{2}x^{\frac{2}{3}} + C$

(e) $\frac{1}{8}x^{\frac{8}{3}} - \frac{3}{7}x^{\frac{7}{4}} + \frac{1}{2}x^{\frac{2}{3}} + C$

3. $\int_{0}^{\frac{\pi}{4}} \frac{1+\sin\theta}{\cos^2\theta}\,\mathrm{d}\theta =$

(a) $2\sqrt{2}$

(b) $\sqrt{2}-1$

(c) $\sqrt{2}$

(d) $\sqrt{2}+1$

(e) $\frac{1}{2}\sqrt{2}$

locity at time t is $v(t) = \sin t$ (measured in meters

rticle during the time period $0 \le t \le \dfrac{3\pi}{2}$ is equal to

18. $\displaystyle\int \frac{2 + \sec x}{2\tan x + x\sec x}\,dx =$

(a) $\dfrac{2}{2}\cos^2 x + 3\sin x + \dfrac{1}{2}x + C$

(b) $\ln|2\tan x + x\sec x| + C$

(c) $\dfrac{\sec x}{\sec x + \tan x} + C$

(d) $\ln|\sin x| + \ln|x| + C$

(e) $\ln|2\sin x + x| + C$

(082)

2. $\displaystyle\int \frac{\cos^2 t}{1+\sin t}\, dt =$

(a) $t + \cos t + c$

(b) $1 + \cos t + c$

(c) $\dfrac{1}{2}t^2 - \cos t + c$

(d) $t - \sin t + c$

(e) $t - \dfrac{1}{2}\sin^2 t + c$

3. $\int (2-\sqrt{x})^2\, dx =$

(a) $4x - \frac{8}{3}\sqrt{x^3} + \frac{1}{2}x^2 + c$

(b) $\frac{(2-\sqrt{x})^3}{3} + c$

(c) $4x + \frac{1}{2}x^2 + c$

(d) $4x - x^{2/3} + \frac{1}{2}x^2 + c$

(e) $4x - 6x^{3/2} + x^2 + c$

9. $\int_{-1}^{2} (x - 2|x|)dx =$

(a) $-\frac{7}{2}$

(b) $-\frac{9}{2}$

(c) $-\frac{5}{}$

(081)

13. $$\int \frac{1}{\sec t - \cos t}\, dt =$$

(a) $\ln|\sin t| + C$

(b) $\cot t + C$

(c) $-\sec t + C$

14. The acceleration (in m/s^2) and the initial velocity for a particle moving along a line are given by

$$a(t) = 2t - 1, v(0) = -2, \quad 0 \leq t \leq 2.$$

The distance traveled by the particle during the given time interval is

(a) $\frac{13}{3}m$

(b) $5m$

(c) $\frac{18}{3}m$

(d) $4m$

(e) $\frac{10}{3}m$

(073)

4. $\displaystyle\int \frac{\cos\theta + \cos\theta\cot^2\theta}{\csc^2\theta} d\theta =$

(a) $\sin\theta + c$

(b) $\tan\theta + c$

(c) $\cot\theta + c$

(d) $\cos\theta + c$

(e) $\csc\theta + c$

10. If a particle moves along a line so that its velocity at time t is $v(t) = 3t - 3$ (measured in meters per second), then the distance (in meters), traveled by the particle during the time period $\frac{1}{2} \le t < 2$ is equal to

(a) $\frac{15}{8}$

(b) $\frac{9}{8}$

(c) $\frac{11}{4}$

(d) $\frac{13}{4}$

(e) $\frac{17}{8}$

(072)

1. The value of $\int_0^{3\pi/4} |\cos x| \, dx$ is equal to

(a) $\frac{1}{9}$

(b) 3

6. $\displaystyle\int_0^{\pi/3} \frac{\sin x}{\cos^2 x} dx =$

(a) -1

(b) $\dfrac{\sqrt{3}}{2} - 1$

(c) 2

(d) 1

(e) $\dfrac{2}{\sqrt{3}} - 1$

8. The value of the integral $\int_0^1 \frac{x^3+x^2+x+1}{x+1}\,dx$ is equal to

(a) 1

(b) $-\frac{2}{3}$

(c) 2

(d) $\frac{4}{3}$

(e) $\frac{7}{3}$

14. Which one of the following statements is **FALSE**?

(a) $e = \lim_{x\to 0}\left(1+\frac{1}{x}\right)^{1/x}$

(b) $\ln x = \int_1^x \frac{1}{t}\,dt \qquad x > 0$

(c) $\frac{d}{dx}(\ln x) = \frac{1}{x}$

(d) e is the number such that $\ln e = 1$

(e) $\int \frac{1}{x}\,dx = \ln|x| + C$

19. A particle moves along a line so that its velocity is $v(t) = 3t^2 - 2t - 8$ (measured in meters per second). Then the **displacement** of the particle during the time period $1 \le t \le 2$ is given by

(a) 0

(b) 4

(c) -6

(d) 6

(e) -4

(071)

7. The value of the integral $\displaystyle\int_1^8 \frac{1+\sqrt[3]{x}}{\sqrt[3]{x^2}}\,dx$ equals

(a) $\dfrac{15}{2}$

(b) $\dfrac{45}{2}$

(c) $\dfrac{7}{2}$

(d) $\dfrac{25}{2}$

(e) $\dfrac{17}{2}$

15. The velocity (in meters per second) of a particle moving along a line is given by $V(t) = 3t^2 - 12t + 9$. The distance traveled between $t = 0$ and $t = 2$ is

(a) 6 meters

(b) 8 meters

(c) 4 meters

(d) 9 meters

(e) 5 meters

16. The integral $\displaystyle\int \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\, d\theta$ is equal to

(a) $-\cos\theta + C$

(b) $\tan\theta + C$

(c) $\cos\theta + C$

(d) $3\sin\theta + C$

(e) $-\sin\theta + C$

(063)

6. $\int_0^{\frac{\pi}{4}} \frac{\sin(2x)}{\sqrt{1-\cos(2x)}} dx =$

(a) 2π

(b) $\frac{1}{2}$

(c) -2

(d) 2

(e) 1

7. $\int_{-2}^{3} |x+1|\, dx =$

(a) $\frac{17}{2}$

(b) $\frac{1}{2}$

(c) 8

(d) $-\frac{17}{2}$

(e) 17

10. If a particle is moving along a straight line and its velocity is given by

$$v(t) = t^2 - 5t + 4, \ 0 \le t \le 4,$$

then its displacement and distance it travels on the time interval $[0, 4]$ are:

(a) displacement $= -\frac{8}{3}$, distance $= \frac{8}{3}$

(b) displacement $= \frac{8}{3}$, distance $= \frac{8}{3}$

(c) displacement $= -\frac{8}{3}$, distance $= \frac{19}{3}$

(d) displacement $= \frac{8}{3}$, distance $= \frac{19}{3}$

(e) displacement $= 0$, distance $= \frac{19}{3}$

(062)

3. The value of $\int_0^1 x(\sqrt[3]{x} + \sqrt[4]{x})dx$ is

(a) $\dfrac{1}{63}$

(b) $\dfrac{1}{16}$

(c) $\dfrac{55}{63}$

(d) $\dfrac{13}{16}$

(e) $\dfrac{11}{63}$

5. The value of $\int_1^2 \frac{6+u+u^2}{u^3} du$ is equal to

(a) $\frac{15}{2}$

(b) $\frac{11}{4} + \ln 2$

(c) $\ln 2 + 4$

(d) $\ln 2 - \frac{5}{6}$

(e) 16

14. The value of $\int_e^{e^2} \frac{dx}{x \ln x}$ is equal to

(a) 1

(b) e

(c) $\ln 2$

(d) $-1 + \ln 2$

(e) $2 \ln 3$

18. The value of $\int_0^{3\pi/2} |\sin x| dx$ is

(a) 1

(b) 2

(c) -1

(d) 3

(e) 0

(061)

1. The integral $\int_1^2 (x + x^{-2})\, dx$ is equal to

(a) 1

(b) 3

(c) $\frac{3}{2}$

(d) $\frac{1}{2}$

(e) 2

12. The integral $\int x(1 + 2x^4)dx$ is equal to

(a) $\frac{x^2}{2} + \frac{x^6}{6} + C$

(b) $1 + 10x^4 + C$

(c) $\frac{x^2}{2}\left(x + \frac{2x^5}{5}\right) + C$

(d) $\frac{x^2}{2} + \frac{x^6}{3} + C$

(e) $\frac{x^2}{2} + \frac{2x^5}{5} + C$

Answer Key :

Question	Answer
4 (092)	A
10 (092)	A
14 (092)	A
2 (091)	A
10 (091)	A
13 (091)	A
15 (091)	A
2 (083)	A
3 (083)	C
13 (083)	A
18 (083)	E
2 (082)	A
3 (082)	A
9 (082)	A
13 (081)	E
14 (081)	E
4 (073)	A
10 (073)	A
1 (072)	D
6 (072)	D
8 (072)	D
14 (072)	A

19 (072)	E
7 (071)	A
15 (071)	A
16 (071)	A
6 (063)	--
7 (063)	--
10 (063)	--
3 (062)	C
5 (062)	B
14 (062)	C
18 (062)	D
1 (061)	E
12 (061)	D

Old Exam 5.5 :

(092)

1. $$\int \frac{e^{2x}}{1+e^{4x}}\,dx =$$

(a) $\frac{1}{2}\tan^{-1}(e^{2x}) + c$

(b) $\tan^{-1}(e^{2x}) + c$

(c) $\frac{1}{4}\tan^{-1}(e^{2x}) + c$

(d) $\frac{1}{2}\tan^{-1}(e^{4x}) + c$

(e) $\tan^{-1}(e^{4x}) + c$

7. If f is an even function such that $\int_{-1}^{1} f(t)\,dt = 5$ and $\int_{-2}^{2} f(t)\,dt = 2$, then $\int_1^2 f(t)\,dt =$

(a) $-\frac{3}{2}$

(b) $\frac{3}{2}$

(c) 3

(d) -3

17. $\int_0^{\frac{\pi}{3}} \sin x \cos 2x\,dx =$

(a) $\frac{1}{12}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

(e) $\frac{1}{6}$

(091)

3. $\displaystyle\int_{e}^{e^3} \frac{1}{x \ln x}\, dx =$

(a) $\ln 3$

(b) $\ln 2$

(c) $1 - \ln 3$

(d) $-\ln 3$

(e) $2 - \ln 2$

12. $$\int_0^1 \frac{10x+15}{\sqrt{2x^2+6x+1}}dx =$$

(a) 10

(b) 5

(c) 20

(d) $\frac{5}{2}$

(e) $\frac{15}{2}$

14. $\displaystyle\int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx =$

(a) $\dfrac{\pi^2}{32}$

(b) $\dfrac{\pi^2}{16}$

(c) $2\pi^2$

(d) $\dfrac{\pi}{8}$

(e) $\dfrac{3\pi}{2}$

16. Which one of the following is **TRUE**: If f is an odd and continuous function on $[-a, a]$, then

(a) $\int_{-a}^{a} [f(x)]^3 dx = 0$

(b) $\int_{-a}^{a} [f(x)]^2 dx = 0$

(c) $\int_{-a}^{a} x f(x) dx = 0$

(d) $\int_{-a}^{a} \cos x \cdot f(x) dx = 2 \int_{0}^{a} \cos x \cdot f(x) dx$

(e) $\int_{-a}^{a} [\sin x + f(x)] dx = 2 \int_{0}^{a} [\sin x + f(x)] dx$

(083)

11. $\int_{1}^{4} \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx =$

(a) $4 \cos e$

(b) $2\left(\cos e - \cos e^2\right)$

(c) $2\left(\sin e^2 - \sin e\right)$

(d) $\frac{1}{2}\left(\cos e - \cos e^2\right)$

(e) $4 \sin e$

17. $\int_1^e \frac{1}{x + x\ln x}\,dx =$

(a) $\ln 2$

(b) $\ln(1+e)$

(c) $\frac{e}{2}$

(d) $2+e$

(e) e

19. $\int_{-\pi}^{\pi} x^5 \cos(x^2)dx =$

(a) $\frac{1}{4} \cos(\pi^2)$

(b) $32 \sin(\pi^2)$

(c) 0

(d) π^6

(e) $4\pi^2 \sin(\pi^2)$

(082)

4. $\displaystyle\int_4^{10} \frac{x}{x^2-4}\,dx =$

(a) $\dfrac{3}{2}\ln 2$

(b) $3\ln 2$

(c) $\dfrac{3}{4}\ln 2$

(d) $\dfrac{1}{2}\ln 2$

(e) $3\ln 4$

10. $\displaystyle\int e^{(2x^5+\ln x^4)}\,dx =$

(a) $\dfrac{1}{10}e^{2x^5}+c$

11. $\int (\sec^2 x) \tan(\tan x)\ dx =$

(a) $\ln|\sec(\tan x)| + c$

(b) $\ln|\tan(\tan x)| + c$

(c) $-\sec(\tan x) + c$

(d) $\ln|\sin(\tan x)| + c$

(e) $\ln(\sec^2 x) + c$

13. $\displaystyle\int_1^2 \frac{x^2 - 2x - 3}{x^4 - 3x^3}\,dx =$

(a) $\dfrac{7}{8}$

(b) $-\dfrac{1}{8}$

(c) $\dfrac{9}{8}$

(d) $-\dfrac{3}{8}$

(e) $\dfrac{5}{8}$

(081)

1. Which one of the following statements is **FALSE**: (f and g are continuous)

(a) If f is even on $[-a, a]$, then $\int_{-a}^{a} f(x)\, dx = 2\int_{-a}^{0} f(x)\, dx$

(b) $\int_{a}^{b}[f(x) - 3g(x)]dx = \int_{a}^{b} f(x)dx - 3\int_{a}^{b} g(x)\, dx$

(c) $\int_{a}^{b} f(x)dx =$ area below the graph of f from $x = a$ to $x = b$.

(d) If $2 \le f(x) \le 6$ on $[0, 3]$, then $6 \le \int_{0}^{3} f(x)\, dx \le 18$.

(e) If $f(x) \le g(x)$ on $[a, b]$, then $\int_{a}^{b} g(x)\, dx \ge \int_{a}^{b} f(x)\, dx$.

4. $\int_{-1}^{1} (3x-2)^{19}\, dx =$

(a) $\dfrac{1-5^{20}}{60}$

(b) 0

(c) $57(1-5^{18})$

(d) $\dfrac{5^{20}}{60}$

(e) $\dfrac{5^{20}-1}{20}$

6. $\displaystyle\int \frac{(x-\sqrt[3]{x})^2}{\sqrt[3]{x^2}} dx =$

(a) $\dfrac{7}{3}x^{7/3} - x^{4/3} + 2x + C$

(b) $\dfrac{3}{7}x^{7/3} - \dfrac{3}{5}x^{5/3} + x + C$

(c) $\dfrac{3}{2}x^{2/3} + \dfrac{6}{5}x^{4/3} + \dfrac{1}{2}x^{2} + C$

(d) $\dfrac{3}{7}x^{7/3} - \dfrac{6}{5}x^{5/3} + x + C$

(e) $\dfrac{1}{3}x^{3} - \dfrac{6}{7}x^{7/3} + \dfrac{3}{5}x^{5/3} + C$

7. $\int e^{x^2+\ln x}\,dx =$

(a) $\dfrac{1}{x}e^{x^2} + C$

(b) $\dfrac{1}{2}e^{x^2} + C$

(c) $\dfrac{e^{x^2+\ln x}}{\left(2x+\frac{1}{x}\right)} + C$

(d) $e^{x^2+\ln x}\left(2x+\dfrac{1}{2}\right) + C$

(e) $e^{x}\ln x + C$

11. By interpreting it as an area, the value of the integral

$$\int_0^1 (|x-1| + 2\sqrt{1-x^2})\, dx$$

is equal to

(a) $\dfrac{\pi+1}{2}$

(b) $2\pi + \dfrac{1}{2}$

(c) $\pi + 1$

(d) $\pi + \dfrac{1}{4}$

(e) $\pi + \dfrac{1}{2}$

15. $\int_{-2}^{2} x^4(xe^{-x^2} + 5)dx =$

(a) 16

(b) -8

(c) 32

(d) 64

(e) 0

19. $\displaystyle\int \frac{x+2}{\sqrt[3]{3-x}}\,dx =$

(a) $-15(3-x)^{1/3} + \dfrac{3}{4}(3-x)^{4/3} + C$

(b) $\ln(3-x) + (3-x) + C$

(c) $\dfrac{5}{3}\ln(3-x) + \dfrac{1}{4}(3-x)^{4/3} + C$

(d) $\dfrac{3}{5}(3-x)^{5/3} + \dfrac{15}{2}(3-x)^{2/3} + C$

(e) $\dfrac{3}{5}(3-x)^{5/3} - \dfrac{15}{2}(3-x)^{2/3} + C$

(073)

1. $\displaystyle\int_1^4 \left(\frac{3\sqrt{x}-5}{\sqrt{x}}\right) dx =$

(a) -1

(b) $\frac{1}{2}$

(c) $-\frac{3}{2}$

(d) $\frac{5}{2}$

(e) $-\frac{1}{2}$

11. $\displaystyle\int \frac{5}{\sqrt{x}(3\sqrt{x}+4)^{3/5}}\, dx =$

(a) $\frac{25}{3}(3\sqrt{x}+4)^{2/5}+c$

(b) $\frac{25}{6}x^{2/5}+\frac{25}{8}x^{1/2}+c$

(c) $\frac{1}{75}(3\sqrt{x}+4)^{2/5}+c$

(d) $\frac{25}{6}x^{2/5}+\frac{25}{16}x^{1/2}+c$

(e) $25(3\sqrt{x}+4)^{2/5}+c$

16. The value of $\displaystyle\int_1^2 x\left(e^{x^2-1}-\frac{1}{2x}\right)\,dx$ is equal to

(a) $\dfrac{1}{2}e^3-1$

(b) $3e^3-1$

(c) $\dfrac{1}{2}e^3-e-1$

(d) $\dfrac{1}{2}e^3-e^2+e-1$

(e) $\dfrac{1}{2}e^3-\ln 2$

18. The value of $\displaystyle\int_{-2}^{2}(x^3+1)\sqrt{4-x^2}\,dx$ is equal to

[Hint: One term of the integral may be interpreted as an area].

(a) 2π

(b) 4π

(c) $3\pi - 2$

(d) $\pi + 2$

(e) $\frac{1}{2}\pi - 4$

11. $\displaystyle\int (\tan x)\ln(\cos x)\,dx =$

(a) $-\dfrac{1}{2}\ln^2\cos x + C$

(b) $\sin x \ln\cos x + C$

(c) $\dfrac{1}{2}(\ln\cos x)^2 + C$

(d) $\dfrac{1}{\cos x}\ln\cos x + C$

(e) $-\ln\cos x + C$

13. $\displaystyle\int_0^1 x(1-x)^{10}\,dx =$

(a) $\dfrac{1}{64}$

(b) $\dfrac{1}{110}$

(c) 1

(d) $\dfrac{1}{11}$

(e) $\dfrac{1}{132}$

17. Evaluate $I = \int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$

(a) 3

(b) 6

(c) $\frac{3}{2}\sqrt[3]{13}$

(d) $\frac{1}{3}$

(e) $\frac{3}{2}$

(071)

3. The integral $\displaystyle\int \frac{\cos\left(\frac{\pi}{x^2}\right)}{x^3}\,dx$ is equal to

(a) $-\dfrac{1}{2\pi}\sin\left(\dfrac{\pi}{x^2}\right)+C$

(b) $\dfrac{1}{2\pi}\sin\left(\dfrac{\pi}{x^2}\right)+C$

(c) $-\dfrac{1}{2\pi}\sin\left(\dfrac{\pi}{x}\right)+C$

(d) $\dfrac{1}{x^2}\sin\left(\dfrac{\pi}{x}\right)+C$

(e) $\dfrac{1}{x^2}\sin\left(\dfrac{\pi}{x^2}\right)+C$

4. The integral $\displaystyle\int \frac{-4x}{\sqrt{1-4x^2}}\,dx$ is equal to

(a) $\sqrt{1-4x^2}+C$

(b) $-\dfrac{1}{4}\sqrt{1-4x^2}+C$

(c) $\dfrac{1}{\sqrt{1-4x^2}}+C$

(d) $\dfrac{-8}{\sqrt{1-4x^2}}+C$

(e) $16\sqrt{1-4x^2}+C$

8. The value of the integral $\int_1^{e^4} \frac{\sqrt{\ln x}}{x}\, dx$ is equal to

(a) $\frac{16}{3}$

(b) $\frac{8}{e^4}$

(c) 8

(d) $\frac{2}{e^4}$

(e) 0

9. The value of the integral $\int_{-3}^{3} \sin(x^5)\, dx$ is equal to

(a) 0

(b) $2\cos(243)$

(c) 6

(d) -3

(e) 1

(063)

1. $\int \frac{3x}{x^2+1} dx =$

(a) $\tan^{-1}(x) + C$
(b) $\frac{1}{3}\tan^{-1}(x) + C$
(c) $\ln(x^2+1) + C$
(d) $\frac{3}{2}\ln(x^2+1) + C$
(e) $\ln|3x| + C$

9. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan(x) dx =$

(a) 1
(b) $\frac{\sqrt{2}}{2}$
(c) $\sqrt{2}$
(d) 0
(e) -1

(062)

6. The value of the integral $\int_{-\pi}^{\pi} \frac{\sin x}{1 + x^2 + x^4}\, dx$ is

(a) 1

(b) 0

(c) -1

(d) 2

(e) -2

7. Let $f(x) = \begin{cases} x & \text{f} \quad -\pi \le x \le 0 \\ \sin x & \text{if} \quad 0 < x \le \pi. \end{cases}$ Then the value of $\int_{-\pi}^{\pi} f(x)dx$ is

(a) $\dfrac{3 - 2\pi^2}{3}$

(b) $\dfrac{\pi^2}{2} - 5$

(c) $\dfrac{8 - \pi^2}{4}$

(d) $2\pi - \pi^2$

(e) $\dfrac{4 - \pi^2}{2}$

8. The integral $\int \frac{1+x}{1+x^2}\,dx$ is equal to

(a) $\tan^{-1} x + \frac{1}{2}\ln(x^2+1) + C$

(b) $1 + \frac{1}{2}\ln(x^2+1) + C$

(c) $\tan^{-1}(x^2+1) + \ln(x^2+1) + C$

(d) $\frac{1}{2}\ln(x^2+1) + C$

(e) $\tan^{-1}(\ln(x^2+1)) + C$

19. The value of $\int_1^e \frac{\cos(\ln x)}{x}\,dx$ is

(a) $\cos 1$

(b) $\cos(\ln 1)$

(c) $\sin(\ln 1)$

(d) $\sin 1$

(e) $\ln(1+e)$

(061)

3. The integral $\displaystyle\int_1^{3/2} \frac{\sin^{-1}(x-1)}{\sqrt{-x^2+2x}}\,dx$ is equal to

(a) $\pi^2/72$

(b) $5/8$

(c) $\pi/72$

(d) $\pi/36$

(e) $\pi^2/36$

8. The integral $\int \frac{x^2}{(x^3+1)^2} dx$ is equal to

(a) $-\frac{1}{x^3+1}+C$

(b) $2\ln(x^3+1)+C$

(c) $6\ln(x^3+1)+C$

(d) $\frac{1}{3(x^3+1)}+C$

(e) $-\frac{1}{3(x^3+1)}+C$

15. The integral $\int_0^{\pi/4} \frac{1+\sin\theta}{\cos^2\theta} d\theta$ is equal to

(a) 2

(b) $\sqrt{2}/2$

(c) $2/\sqrt{2}$

(d) $3/\sqrt{2}$

(e) $\sqrt{2}/3$

Answer Key :

Question	Answer
1 (092)	A
7 (092)	A
17 (092)	A
3(091)	A
12 (091)	A
14(091)	A
16 (091)	A
11(083)	C
17(083)	A
19 (083)	C
4 (082)	A
10 (082)	A
11(082)	A
13 (082)	A
1 (081)	C
4 (081)	A
6 (081)	D
7(081)	B
11 (081)	A
15(081)	D

19 (081)	E
1 (073)	A
11 (073)	A
16 (073)	A
18 (073)	A
11 (072)	A
13(072)	E
17 (072)	A
1 (063)	--
9 (063)	--
6(062)	B
7 (062)	E
8(062)	A
19(062)	D
3(061)	A
8 (061)	E
15 (061)	C

Review Chapter 5 .

Review Chapter 5：

(092)

23. $\displaystyle\int_{-1}^{1} \frac{1+\tan x + x^2}{1+x^2}\,dx =$

(a) 2

(b) 0

(c) 1

(d) 3

(e) 4

(091)

3. If $F(x) = \int_{\sqrt{x}}^{\sqrt{2}} \frac{2}{1+t^4}\,dt$, then $F'(x) =$

(a) $\frac{-1}{(1+x^2)\sqrt{x}}$

(b) $\frac{2}{5} - \frac{2}{1+x^2}$

(c) $\frac{2}{1+x^2}$

(d) $\frac{2}{1+x^4}$

(e) $\frac{2}{(1+x^2)\sqrt{x}}$

4. $\lim_{n\to+\infty} \sum_{i=1}^{n} \frac{1}{n} \cdot e^{-\frac{2i}{n}} =$

(a) $\int_0^2 \frac{1}{2} e^{-x} dx$

(b) $\int_0^2 e^{-2x} dx$

(c) $\int_0^1 2e^{-x} dx$

(d) $\int_0^1 e^{-\frac{x}{2}} dx$

(e) $\int_1^2 2e^{-x} dx$

20. $\int_0^1 |4x-3|\,dx =$

(a) $\frac{5}{4}$

(b) 7

(083)

8. If $F(x) = \int_1^{x^3} \tan^{-1}(\sqrt[3]{t})\, dt$, then $F(1) + F'(1) + F''(1) =$

(a) $\frac{3\pi}{2} - \frac{1}{2}$

(b) $\frac{9\pi}{4}$

(c) $\frac{9\pi}{4} + \frac{3}{2}$

(d) $\frac{3\pi}{2} + \frac{3}{2}$

(e) $\frac{7\pi}{4} + \frac{1}{2}$

10. If $f(x) = \begin{cases} \sqrt{3-x} & \text{if } x \leq 2 \\ e^{x-2} & \text{if } x > 2, \end{cases}$

then $\int_{-1}^{3} f(x)dx =$

(a) $e + \frac{11}{3}$

(b) $\frac{1}{2}e + \frac{4}{3}$

(c) $e + \sqrt{2}$

(d) $2e - \frac{14}{3}$

(e) $e - 8$

26. $\int_0^1 \frac{d}{dx}\left(\frac{e^x}{x^2+1}\right) dx + \frac{d}{dx}\int_0^1 \frac{e^x}{x^2+1}\, dx =$

(a) $e + 1$

(b) $\frac{1}{2}e + 3$

(c) can not be evaluated

(d) $\frac{e-2}{2}$

(e) $e - \frac{1}{2}$

(081)

5. If f is a continuous function and $F(x) = \int_1^{x^3} f(\sqrt[3]{t})\, dt$, then $F'(x) =$

(a) $3x^2\, f(\sqrt[3]{x})$

(b) $\frac{1}{3}\, x^{-2/3}\, f(\sqrt[3]{x})$

(c) $\frac{1}{3x^2}\, f(x)$

(d) $3x^2\, f(x)$

(e) $f(x) - f(1)$

21. The value of the integral $\int_{-1}^{1} \frac{4 - x|x|}{2 + x}\, dx$ is equal to

(a) $5 + \ln 2$

(b) $-1 + 8\ln 2$

(c) $-4 + \ln 2$

(d) $4 + 8\ln 2$

(e) -2

(073)

6. $\int_0^{\pi} |\sin 2x| \ dx =$

(a) 2

(b) 0

(c) -1

(d) 1

(e) $\frac{1}{2}$

16. If $x > e$, then $\dfrac{d}{dx}\int_1^{\sqrt{\ln x}} e^{t^2}\,dt =$

(a) $\dfrac{1}{2\sqrt{\ln x}}$

(b) $\dfrac{4}{\sqrt{\ln x}}$

(c) 0

(d) x

(e) $2x$

19. The value of the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\dfrac{\pi}{4n}\left(\cos\dfrac{i\pi}{2n}\right)^2$ on the interval $\left[0,\dfrac{\pi}{2}\right]$ is

(a) $\dfrac{\pi}{8}$

(b) $\dfrac{\pi}{2}$

(c) $1+\dfrac{\pi}{8}$

(d) $1+\dfrac{\pi}{2}$

(e) $-\dfrac{1}{4}+\dfrac{\pi}{8}$

25. If $\int_0^{\pi} x\, f(\sin x)\, dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x)\, dx$, then $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx =$

(a) $\frac{\pi^2}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi^2}{16}$

(d) π

(e) π^2

5. If f' is continous function, $f(1) = 3$, and $\int_0^3 x f'(1+x^2)dx = 4$, then $f(10) =$

(a) 9

(b) 11

(c) 8

(d) 10

(e) 5

10. $\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{1+(\frac{i}{n})^2} =$

(a) $\frac{1}{4}$

(b) ∞

(c) $\frac{\pi}{4}$

(d) π

(e) $\frac{\pi}{2}$

21. If $I = \int_{-1}^{1} \sin(x^2)dx$, then

(a) $0 \le I \le 2$

(b) $I = \infty$

(c) $I = 0$

(d) $I > 2$

(e) $I \le 0$

(071)

2. If $y = \int_{1-3x}^{1} \frac{u^3}{1+u^2} du$, then $\frac{dy}{dx} =$

(a) $\frac{3(1-3x)^3}{1+(1-3x)^2}$

(b) $\frac{-3(1-3x)^3}{1+(1-3x)^2}$

(c) $\frac{(1-3x)^3}{1+(1-3x)^2}$

(d) $\frac{27x^3}{1+9x^2}$

(e) $\frac{81x^3}{1+9x^2}$

(063)

13. If $\int_2^3 f(x)dx = 3\int_1^2 f(x)dx$, $\int_1^2 f(x)dx + \int_3^4 f(x)dx = \int_2^3 f(x)dx$, $\int_1^5 f(x)dx = 17$ and $\int_2^3 f(x)dx = 4$, then $\int_4^5 f(x)dx$ equals

(a) 9

(b) 3

(c) 4

(d) -9

(e) -3

(062)

7. By recognizing the sum as a Riemann sum for a function defined on $[0,1]$, the value of the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}e^{-i/n}$ is

(a) 0

(b) $e-1$

(c) 1

(d) -3

(e) $1-e^{-1}$

12. If $F(x) = \int_3^{x^2} \frac{\tan^{-1}\sqrt{t}}{\sqrt{t}}\, dt,\ \ x > 0$, then $8F(\sqrt{3}) + 9F'(\sqrt{3}) =$

(a) 6π

(b) 8π

(c) $17\sqrt{3}\pi$

(d) 3π

(e) $\sqrt{3}\pi$

(061)

9. The limit $\lim\limits_{n\to+\infty} \sum\limits_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}}$ represents the area of the region under the graph of

(a) $y = \sqrt{x}$ on the interval $[0, 3]$
(b) $y = \sqrt{x+1}$ on the interval $[1, 4]$
(c) $y = \sqrt{x}$ on the interval $[1, 4]$
(d) $y = \sqrt{x+1}$ on the interval $[0, 4]$
(e) $y = \sqrt{x+1}$ on the interval $[1, 3]$

10. Use the properties of the integrals to determine which of the following relations is **correct**

(a) $\int_1^3 \sqrt{x^4+1}dx \geq \frac{26}{3}$

(b) $2 < \int_1^3 \sqrt{x^4+1}dx < 2\sqrt{2}$

(c) $\int_1^3 \sqrt{x^4+1}dx < 0$

(d) $2\sqrt{2} < \int_1^3 \sqrt{x^4+1}dx < \frac{26}{3}$

(e) $0 < \int_1^3 \sqrt{x^4+1}dx \leq 2$

11. The integral $\int_0^2 |x - x^2| \, dx$ is equal to

(a) $-2/3$
(b) 1
(c) -1
(d) $5/6$
(e) $2/3$

Answer Key :

Question	Answer

23 (092)	A
3(091)	A
4 (091)	A
8(083)	C
10(083)	A
26 (083)	B
5 (081)	B
21 (081)	D
6 (073)	A
16 (073)	A
19 (073)	A
25 (073)	A
5 (072)	B
10(072)	C
21 (072)	A
2 (071)	A
13 (063)	--
7(062)	E
12(062)	A
9(061)	C
10(061)	A
11(061)	B

Old Exam 6.1 :

(092)

12. The area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \frac{\pi}{2}$ is equal to

(a) $2\sqrt{2} - 2$

(b) $4\sqrt{2} + 2$

(c) $2\sqrt{2} + 2$

(d) 4

(e) $\sqrt{2} - 1$

13. The area of the region enclosed by the curves, $y = x^2 - 4$, $y = -2x + 4$, and $y = -4$ is equal to

(a) $\frac{20}{3}$

(b) $\frac{17}{3}$

(c) $\frac{8}{5}$

(d) $\frac{12}{5}$

(e) 3

(091)

6. The area of the region enclosed by the graphs of

$$2y^2 = x + 4 \text{ and } x = y^2$$

is equal to

(a) $\frac{32}{3}$

(b) $\sqrt{3}$

(c) $4\sqrt{2}$

(d) $\frac{1}{2}$

(e) 1

17. The area of the region lying between the curves $y = x^2$ and $y = -x + 2$ and between the lines $x = 0$ and $x = 2$ is equal to

(a) 3

(b) 2

(c) $\frac{5}{2}$

(d) $\frac{7}{3}$

(e) $\frac{3}{5}$

(083)

6. The area of the region bounded by the curves $y^2 - x = 4$ and $y^2 + x = 2$ is equal to

(a) 4

(b) 6

(c) $4\sqrt{3}$

(d) $8\sqrt{3}$

(e) 3

10. The area of the region between the curves $y = \sin x$ and $y = \frac{1}{2}$ from $x = 0$ to $x = \frac{\pi}{2}$ is equal to

(a) $\sqrt{2} - \frac{\pi}{12} + 2$

(b) $\sqrt{3} + \frac{\pi}{12} - 1$

(c) $\sqrt{3} - \frac{\pi}{12} - 1$

(d) $\sqrt{3} - \frac{\pi}{6} - 2$

(e) $\sqrt{2} - \frac{\pi}{2} + 1$

(082)

16. The area enclosed by the line $x + 2y = 1$ and the parabola $y^2 = 4 - x$ is given by the definite integral

(a) $\int_{-1}^{3} (3 + 2y - y^2)dy$

(b) $\int_{-1}^{3} \left[\frac{1}{2}(1 - x) - \sqrt{4 - x}\right] dx$

(c) $\int_{-1}^{3} (y^2 - 2y - 3)dy$

(d) $\int_{-3}^{4} \left[\frac{1}{2}(1 - x) - \sqrt{4 - x}\right] dx$

(e) $\int_{-3}^{1} (3 - 2y + y^2)dy$

18. The area of the region in the right half of the plane bounded by the curves $y = 2x - 1$, $y = x^2$, and $y = -x$ is equal to

(a) $\int_{0}^{1/3} (x^2 + x)dx + \int_{1/3}^{1} (x^2 - 2x + 1)dx$

(b) $\int_{0}^{1} (x^2 - x + 1)dx$

(c) $\int_{0}^{1/3} (x^2 - x)dx + \int_{1/3}^{1} (x^2 + 2x - 1)dx$

(d) $\int_{0}^{1} (x^2 + x - 1)dx$

(e) $\int_{0}^{1/3} (x - x^2)dx + \int_{1/3}^{1} (x^2 - 2x - 1)dx$

(081)

10. If the line $x = k$ divides the region bounded by the curves $y = \sqrt{x}, y = 0$ and $x = 4$ into two regions with equal area, then $k =$

(a) $\sqrt[3]{16}$

(b) 4

(c) 8

(d) $\sqrt[3]{4}$

(e) 2

16. The area of the region bounded by the curves $x = -2y^2$ and $y = x + 1$ is

(a) $\frac{5}{24}$

(b) $\frac{5}{8}$

(c) $\frac{27}{8}$

(d) $\frac{1}{24}$

(e) $\frac{9}{8}$

(073)

13. The area enclosed by the line $2x+y=1$ and the parabola $y=4-x^2$ is given by the definite integral

(a) $\int_{-1}^{3}(3+2x-x^2)\,dx$

(b) $\int_{-5}^{3}\left(\frac{1}{2}(1-y)-\sqrt{4-y}\right)\,dy$

(c) $\int_{-1}^{3}(x^2-2x-3)\,dx$

(d) $\int_{-5}^{4}\left(\frac{1}{2}(1-y)-\sqrt{4-y}\right)\,dy$

(e) $\int_{-3}^{1}(3-2x+x^2)\,dx$

19. The area enclosed by the graphs of $y=\sin x$, $y=\sin 2x$, $x=0$ and $x=\frac{\pi}{2}$ is equal to

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{3}{2}$

(d) $\frac{3}{4}$

(e) $\frac{3}{8}$

(072)

15. The area between the curves of $y = x^2 - 1$ and $y = x + 1$ is

(a) 5

(b) 9

(c) $\frac{9}{4}$

(d) 3

(e) $\frac{9}{2}$

18. Find the area of the region bounded by the graphs of the equations

$$x = 2y^2 \quad \text{and} \quad y^2 = \frac{x}{3} + 3$$

(a) 36

(b) 72

(c) $\frac{1}{2}\sqrt{6} - 8 + 3\sqrt{3}$

(d) 18

(e) $\sqrt{6} - 16 + 6\sqrt{3}$

(071)

5. The area of the region bounded by the curves $y = e^x, y = x, x = 0$ and $x = 1$ is

(a) $e - \frac{3}{2}$

(b) $e + \frac{1}{2}$

(c) e

(d) $3 - e$

(e) $e + 1$

11. The area of the region enclosed by the curves $y = \sin x, y = \cos x, x = 0$ and $x = \pi$ is

(a) $2\sqrt{2}$

(b) $2\sqrt{2} - 1$

(c) $2\sqrt{2} + 1$

(d) $\sqrt{2} + 2$

(e) $-\sqrt{2}$

11. The area of the region enclosed by the graphs of $x = y + 1$ and $x = (y-1)^2$ is equal to

(a) 6

(b) $\frac{9}{2}$

(c) 4

(d) 5

(e) $\frac{11}{2}$

12. The area of the region enclosed by the graphs of $y = |x|$ and $y = 2 - x^2$ is equal to

(a) $\int_{-1}^{0}(2 + x - x^2)dx + \int_{0}^{1}(2 - x - x^2)dx$

(b) $\int_{-\sqrt{2}}^{\sqrt{2}}(2 - x^2 - x)dx$

(c) $\int_{-1}^{1}(2 - x^2 - x)dx$

(d) $\int_{-1}^{0}(2 - x - x^2)dx + \int_{0}^{1}(2 + x - x^2)dx$

(e) $\int_{-1}^{1}(2 - x^2 + x)dx$

16. The area of the region enclosed by the graphs of $y = \sin x$, $y = \sin 2x$, $x = 0$ and $x = \dfrac{\pi}{3}$ is equal to

(a) $\sqrt{2} - 1$

(b) $\sqrt{3} - \dfrac{3}{2}$

(c) $\dfrac{1}{4}$

(d) $\dfrac{1}{2}$

(e) 0

(061)

4. The area enclosed by the graphs of $y = x^3 - x$ and $y = 3x$ is equal to

(a) 0

(b) $7/2$

(c) 4

(d) 2

(e) 8

5. The area enclosed by the graphs of $y = \sin x$ and $y = \sin 2x$, $x = 0$ and $x = \dfrac{\pi}{2}$ is equal to

(a) $\dfrac{1}{4}$

(b) 0

(c) $\dfrac{1}{2}$

(d) 1

(e) 2

10. The area enclosed by the graphs of $x = y + 2$ and $x = y^2$ is equal to

(a) 4

(b) 3

(c) $\frac{9}{2}$

(d) $\frac{16}{3}$

(e) $\frac{7}{6}$

Answer Key :

Question	Answer
12 (092)	A
13(092)	A
6(091)	A
17 (091)	A
6 (083)	D

10(083)	C
16 (082)	A
18 (082)	A
10 (081)	A
16 (081)	E
6 (081)	D
7(081)	B
15 (073)	A
19 (073)	A
15 (072)	E
18(072)	A
5(071)	A
11 (071)	B
11(062)	A
12(062)	C
4(061)	E
5(061)	C
10(061)	C

Old Exam Chapter 6.2 & 6.3 :

(092)

6. The volume of the solid resulting from the region: $y = -x^2 + 6x - 8$; $y = 0$ which has been rotated about the $y-$ axis is given by the definite integral:

(a) $\int_2^4 2\pi\, x \left[-x^2 + 6x - 8\right] \, dx$

(b) $\int_2^4 \pi\, x \left[-x^2 + 6x - 8\right] \, dx$

(c) $\int_0^8 2\pi\, x \left[-x^2 + 6x - 8\right] \, dx$

(d) $\int_2^4 2\pi \left[-x^2 + 6x - 8\right] \, dx$

(e) $\int_0^4 2\pi\, x \left[-x^2 + 6x - 8\right] \, dx$

11. If the region enclosed by the curves $y = x$ and $y = x^3$, where $x \geq 0$, is rotated about the $x-$axis, then the volume of the solid obtained is equal to

(a) $\dfrac{4\pi}{21}$

(b) $\dfrac{\pi}{4}$

(c) $\dfrac{11\pi}{21}$

(d) $\dfrac{7\pi}{21}$

(e) $\dfrac{\pi}{7}$

18. Using cylindrical shells, the volume of the solid that is generated when the region enclosed by $y = x^3$, $y = 1$, $x = 0$ is revolved about $y = 1$, is

(a) $\dfrac{9\pi}{14}$

(b) $\dfrac{7\pi}{15}$

(c) $\dfrac{15\pi}{21}$

(d) $\dfrac{3\pi}{14}$

(e) $\dfrac{17\pi}{14}$

19. The volume of the solid whose base is the region bounded between the curves $y = x$ and $y = x^2$, and whose cross sections perpendicular to the $x-$ axis are squares is

(a) $\dfrac{1}{30}$

(b) $\dfrac{1}{12}$

(c) $\dfrac{1}{18}$

(d) $\dfrac{1}{36}$

(e) $\dfrac{1}{24}$

(091)

5. The volume of the solid generated by rotating the region bounded by the curves

$$x^2 + y^2 = 1 \text{ and } y = |x|$$

about the x-axis is equal to

(a) $\dfrac{2\sqrt{2}}{3}\pi$

(b) $\dfrac{2}{3}\pi$

(c) $4\sqrt{2}\pi$

(d) $\sqrt{3}\pi$

(e) $\dfrac{2}{\sqrt{3}}\pi$

18. The volume of the solid generated by rotating the region bounded by the curves

$$y = x \text{ and } y = \sqrt{x}$$

about the line $x = 2$ is given by

(a) $\int_0^1 \pi[(2-y^2)^2 - (2-y)^2]dy$

(b) $\int_0^1 \pi[(2-\sqrt{x})^2 - (2-x)^2]dx$

(c) $\int_0^1 \pi[(y+2)^2 - (y^2+2)^2]dy$

(d) $\int_0^1 \pi[(\sqrt{x}+2)^2 - (x+2)^2]dx$

(e) $\int_0^1 \pi(y^2 - y - 2)dy$

(083)

12. The volume of the solid obtained by rotating the region bounded by the curves $y = x^3$, $y = 1$, and $x = 0$ about the y-axis is equal to

(a) $\frac{3\pi}{7}$

(b) $\frac{\pi}{5}$

(c) $\frac{3\pi}{4}$

(d) $\frac{2\pi}{3}$

(e) $\frac{3\pi}{5}$

16. The volume of the solid generated by revolving the region bounded by the parabolas $y = x^2$ and $y^2 = 8x$ about the line $y = -1$ is given by

(a) $\pi \int_0^2 \left(8x - x^4\right) dx$

(b) $\pi \int_0^2 \left[\left(\sqrt{8x} + 1\right)^2 - \left(x^2 + 1\right)^2\right] dx$

(c) $\pi \int_0^{16} \left[\left(\sqrt{y} + 1\right)^2 - \left(\frac{1}{8} y^2 + 1\right)^2\right] dy$

(d) $\pi \int_0^{16} \left[\left(\sqrt{y} - 1\right)^2 - \left(\frac{1}{8} y^2 - 1\right)^2\right] dy$

(e) $\pi \int_0^2 \left(\sqrt{8x} - x^2\right)^2 dx$

20. A solid has a base lying in the first quadrant and is bounded by the curves $y = 1 - \frac{1}{4}x^2$, $x = 0$, and $y = 0$. If the cross sections of the solid perpendicular to the x-axis are squares, then the volume of the solid is equal to

(a) $\frac{16}{15}$

(b) $\frac{8}{15}$

(c) $\frac{14}{15}$

(d) $\frac{11}{15}$

(e) $\frac{17}{15}$

14. The volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y^2 = x$ about the x-axis is equal to

(a) $3\pi/10$

(b) $37\pi/10$

(c) $\pi/10$

(d) $\pi/6$

(e) $5\pi/6$

15. The base of a solid is the region s bounded by the curves $y = \sqrt{x}, y = 0, x = 1$ and $x = 2$. If the cross-sections of the solid perpendicular to the x-axis are squares with one side lying along the base, then the volume of the solid is

(a) $\frac{3}{2}$

(b) 2

(c) 1

(d) $\frac{1}{2}$

(e) $\frac{2}{3}(2\sqrt{2} - 1)$

17. The volume of the solid obtained by rotating the region bounded by the curves $y = \frac{1}{x}, y = 0, x = 1$ and $x = 3$ about the line $y = 1$ is

(a) $2\pi\left(\ln 3 - \frac{1}{3}\right)$

(b) $\pi\left(\ln 3 + \frac{1}{3}\right)$

(c) $2\pi\left(\ln 3 - \frac{2}{3}\right)$

(d) $\frac{2\pi}{3}$

(e) $\pi\left(2\ln 3 - \frac{1}{3}\right)$

(081)

12. The volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}$, $1 \leq y \leq 4$, about the y-axis is equal to

(a) 3π

(b) π

(c) $\frac{6\pi}{7}$

(d) 10π

(e) -3π

17. A solid has a circular base of radius 1 and center $(0, 0)$. If the cross-sections of the solid perpendicular to the x-axis are semicircles, then the volume of the solid is equal to

(a) $\frac{2\pi}{3}$

(b) $\frac{16\pi}{3}$

(c) $\frac{8\pi}{3}$

(d) $\frac{\pi}{3}$

(e) $\frac{4\pi}{3}$

18. If the region enclosed by the curves $y = x^2$ and $y = 2x$ is rotated about the line $y = 5$, then the volume of the resulting solid is given by

(a) $\pi \int_0^2 [(5 - x^2)^2 - (5 - 2x)^2] \, dx$

(b) $\pi \int_0^4 \left[(\sqrt{y})^2 - \left(\frac{1}{2} \, y \right)^2 \right] \, dy$

(c) $\pi \int_0^2 [(2x + 5)^2 - (x^2 + 5)^2] \, dx$

(d) $\pi \int_0^2 [(2x)^2 - (x^2)^2] \, dx$

(e) $\pi \int_0^4 \left[(5 - \sqrt{y})^2 - \left(5 - \frac{1}{2} \, y \right)^2 \right] \, dy$

(073)

14. The volume of the solid obtained by rotating the region bounded by the graphs of $y^2 = x$ and $y = x - 2$, about the y-axis is given by the definite integral

(a) $\pi \int_{-1}^{2} (4 + 4y + y^2 - y^4)\, dy$

(b) $\pi \int_{1}^{4} (x^2 - 3x + 4)\, dx$

(c) $\pi \int_{-1}^{2} (4 - 4y - y^2 - y^4)\, dy$

(d) $\pi \int_{1}^{4} (x^2 + 3x - 4)\, dx$

(e) $\pi \int_{-2}^{1} (4 - 4y + y^2 - y^4)\, dy$

15. The volume of the solid obtained by rotating the region bounded by the graphs of $y = -\frac{1}{x}$, $y = \frac{1}{x}$, $x = 1$, and $x = 2$, about $y = 1$, is equal to

(a) $4\pi \ln 2$

(b) $4\pi \ln 2 + 1$

(c) $\pi \ln 2$

(d) $\pi \ln 2 + 4$

(e) $8\pi \ln 2$

20. Let S be a solid whose base is enclosed by the graphs of $y = x^2$ and $y = 1$ and whose cross-sections perpendicular to the y-axis are squares. Then the volume of S is

(a) 2

(b) 4

(c) $\frac{3}{2}$

(d) $2\sqrt{2}$

(e) 8

(072)

12. If the region enclosed by the curves $y = x$ and $y = x^2$ is rotated about the line $x = -1$, then the volume of the solid obtained is equal to

(a) $\frac{\pi}{4}$

(b) $\frac{2\pi}{3}$

(c) $\frac{2\pi}{15}$

(d) $\frac{\pi}{3}$

(e) $\frac{\pi}{2}$

16. The volume generated by rotating the region bounded by $y = \ln x$, $y = 0$, and $x = e$ about the y-axis is equal to

(a) $\pi \int_1^e (\ln x)^2 dx$

(b) $\pi \int_0^1 (e - e^y)^2 dy$

(c) $\pi \int_1^e (e - e^y)^2 dy$

(d) $\pi \int_0^1 (e^2 - e^{2y}) dy$

(e) $\pi \int_0^e (e^2 - e^{2y}) dy$

20. The region bounded by the graphs of the equations $2x - y = -1$ and $y = 5x^2 + 2$ and by the vertical lines $x = 0$ and $x = 1$ is revolved about the x-axis. Find the volume of the resulting solid

(a) $\dfrac{34\pi}{3}$

(b) $\dfrac{34\pi}{6}$

(c) $\dfrac{10\pi}{3}$

(d) $\dfrac{5\pi}{3}$

(e) $\dfrac{5\pi}{6}$

(071)

19. The base of a solid S is enclosed by the curves $y = x^2, y = 0$ and $x = 2$. If the cross-sections of S perpendicular to the x-axis are squares, then the volume of S is equal to

(a) $\dfrac{32}{5}$

(b) $\dfrac{16}{3}$

(c) $\dfrac{19}{4}$

(d) 13

(e) 4

20. If the region enclosed by the curves $y = \dfrac{1}{2}x$ and $y = \sqrt{x}$ is rotated about the line $x = -1$, then the volume of the solid is given by

(a) $\pi \displaystyle\int_0^2 [(2y+1)^2 - (y^2+1)^2]dy$

(b) $\pi \displaystyle\int_0^2 [(2y-1)^2 - (y^2-1)^2]dy$

(c) $\pi \displaystyle\int_0^4 \left[\left(\frac{1}{2}x+1\right)^2 - (\sqrt{x}+1)^2\right] dx$

(d) $\pi \displaystyle\int_0^4 \left[\left(\frac{1}{2}x-1\right)^2 - (\sqrt{x}-1)^2\right] dx$

(e) $\pi \displaystyle\int_0^2 [2y - y^2 - 1]^2 dy$

(062)

2. The volume of the solid obtained by rotating the region bounded by the curves
$y = \sqrt{x-1}$, $y = 0$, and $x = 5$ about x-axis is equal to

(a) 10π

(b) 4π

(c) 6π

(d) 8π

(e) 2π

10. The integral $\int_0^{\pi/2} \pi[(1+\cos x)^2 - 1^2]\,dx$ represents the volume of the solid obtained by rotating the region bounded by

(a) $y = 1+\cos x,\ y = 0,\ x = 0,$ and $x = \pi/2$ about the x-axis

(b) $y = 1+\cos x,\ y = 1,\ x = 0,$ and $x = \pi/2$ about the x-axis

(c) $y = (1+\cos x)^2,\ y = 1,\ x = 0,$ and $x = \pi/2$ about the x-axis

(d) $y = 1-\cos x,\ y = 1,\ x = 0,$ and $x = \pi/2$ about the x-axis

(e) $y = 2+\cos x,\ y = 1,\ x = 0,$ and $x = \pi/2$ about the x-axis

17. The volume of the solid obtained by rotating the region bounded by the curves
$y = x^2$, and $x = y^2$ about the line $x = 1$ is equal to

(a) $\pi \int_0^1 (y^2 - y)dy$

(b) $\pi \int_0^1 ((1 - y^2)^2 - (1 - \sqrt{y})^2)dy$

(c) $\pi \int_0^1 ((1 - y^2)^2 - (1 - y)^2)dy$

(d) $\pi \int_0^1 ((1 - \sqrt{y})^2 - (1 - y)^2)dy$

(e) $\pi \int_0^1 (y^2 - \sqrt{y})^2 \, dy$

(061)

1. The volume of the solid obtained by rotating about the y-axis the region bounded by: $y = \frac{1}{x}$, $y = 0$, $x = 1$ and $x = 2$ is equal to

(a) 3π
(b) 2π
(c) $\pi/3$
(d) π
(e) $2\pi \ln 2$

2. The volume of the solid obtained by rotating the region bounded by $y = 1 - x^2$ and $y = 0$, about $y = -1$ is equal to

(a) $43\pi/5$
(b) $28\pi/15$
(c) $56\pi/15$
(d) $86\pi/5$
(e) $12\pi/5$

3. The volume generated by rotating the region bounded by $y = \ln x$, $y = 0$, and $x = e$ about the y-axis is equal to

(a) πe
(b) $\pi e(e - 1)$
(c) π
(d) $\pi e - \pi + 1$
(e) $\frac{\pi}{2}(e^2 + 1)$

Answer Key :

Question	Answer

6 (092)	A
11(092)	A
18(092)	A
19 (092)	A
5 (091)	A
18(091)	A
12 (083)	E
16 (083)	B
20 (083)	A
14 (082)	A
15 (082)	A
17(082)	A
12 (081)	A
17 (081)	A
18 (081)	A
14(073)	A
15(073)	A
20 (073)	A
12(072)	E
16(072)	A
20(072)	A
19(071)	A
20(071)	A
2(062)	D
10(062)	B
17(062)	B

1(061)	E
2(061)	B
3(061)	A

Old Exam 6.5 :

(092)

Q.4. $(8-\ points)$.*Find* all possible values of the number b such that the average value of $f(x) = 4 + 8x - 3x^2$ on the interval $[0, b]$ is equal to 3.

(091)

2. [**5 points**] Find the average value of the function $f(t) = \tan t \sec t$ over the interval $\left[0, \dfrac{\pi}{4}\right]$.

(083)

3. [**10 points**] Find the average value of the function $f(x) = (\sin^{-1} x)^2$ on the interval $[0, 1]$.

(082)

Q.3. (4 points). Find the average value of the function $f(t) = t \sin(t^2)$ on the interval $[0, \sqrt{\pi}]$.

(081)

2. **[7 points]** Find the average value of the function $f(x) = x\sec^2(2x)$ on the interval $\left[0, \frac{\pi}{8}\right]$.

(072)

3. (6 points) Find the average value of the function $f(x) = x\tan^{-1} x$ on the interval $[-1, 1]$.

(071)

2. Find all numbers b such that the average value of $f(x) = \sqrt{x}$ on the interval $[0, b]$ is 6. (7 points)

(063)

9. The average value of $f(x) = \dfrac{1}{1-\cos x}$ on $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$ equals

(a) $\dfrac{12(1+\sqrt{2}-\sqrt{3})}{\pi}$

(b) $\dfrac{12(1-\sqrt{2}+\sqrt{3})}{\pi}$

(c) $\dfrac{12(1-\sqrt{2}-\sqrt{3})}{\pi}$

(d) $\dfrac{\pi(1+\sqrt{2}-\sqrt{3})}{12}$

(e) $12\pi(1+\sqrt{2}-\sqrt{3})$

(062)

1. For $f(x) = 3\sqrt{x+1}, \quad -1 \le x \le 8$

(a) Find the average value of f on the given interval.

(b) Using part (a) find a number c which satisfies the mean value theorem for integrals.

(061)

8. The average value of $\frac{\left(\tan^{-1} x\right)^2}{1+x^2}$ on $[0, 1]$ is equal to

(a) $\pi^3/192$
(b) $3\pi^2/25$
(c) $3\pi^2/16$
(d) $3\pi^2/4$
(e) $\pi^3/25$

Review Chapter 6 .

Review Chapter 6 :

(092)

8. If the region bounded by the curves $y = 4(x-1)^2$ and $y = 4$ is revolved about the line $x = -1$, then the volume of the solid generated is given by

(a) $\int_0^2 2\pi(8x - 4x^2)(x + 1)\, dx$

(b) $\int_{-1}^2 2\pi(8x - 4x^2)(x + 1)\, dx$

(c) $\int_0^2 16\pi(x - 1)^4\, dx$

(d) $\int_0^2 8\pi(x - 1)^2(x + 1)\, dx$

(e) $\int_{-1}^2 8\pi(x - 1)^2(x + 1)\, dx$

9. The volume of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = \sqrt{x}$ about the line $y = 1$, is

(a) $\dfrac{\pi}{6}$

(b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{2}$

(d) π

(e) $\dfrac{\pi}{4}$

10. The average value of the function $f(x) = \dfrac{1}{2}\sin x \sin 2x$ on the interval $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{4}\right]$ is

(a) $\dfrac{\sqrt{2}+4}{9\pi}$

(b) $\dfrac{\sqrt{2}-4}{9\pi}$

(c) $\dfrac{\sqrt{2}+4}{3\pi}$

(d) $\dfrac{\sqrt{3}+4}{3\pi}$

(e) $\dfrac{2\sqrt{3}+4}{3\pi}$

24. The area of the triangle bounded by the lines

$$y = x, \ y = -3x \quad \text{and} \ y = -x + 2$$

is equal to

(a) 2

(b) 3

(c) 4

(d) $\frac{1}{2}$

(e) $\frac{4}{3}$

(083)

1. The volume of the solid generated by revolving the region bounded by the curves

$$y = x^3, x = 0, \text{ and } y = 1$$

about the line $y = 2$ is given by

(a) $V = \int_0^1 \pi \cdot [(2 - x^3)^2 - 2^2]dx$

(b) $V = \int_0^1 \pi[(\sqrt[3]{y})^2 - 1]dy$

(c) $V = \int_0^1 \pi \cdot (x^6 - 1)dx$

(d) $V = \int_0^1 \pi \cdot [(2 - x^3)^2 - 1]dx$

(e) $V = \int_0^1 \pi[(2 - \sqrt[3]{y})^2 - 1]dy$

6. The area of the region enclosed by the curves

$$y = x^2 - 1 \text{ and } y = x + 1$$

is equal to

(a) 2

(b) $\frac{3}{2}$

(c) 4

(d) $\frac{9}{2}$

(e) -3

13. The volume of the solid obtained by rotating the region enclosed by the curves

$$y = \frac{x}{1+x^6}\ , \quad y = 0, x = 0, \text{ and } x = 1$$

about the y-axis is equal to

(a) $\frac{\pi^2}{12}$

(b) $\frac{\pi}{12}$

(c) $\frac{\pi^2}{6}$

(d) $\frac{2\pi}{3}$

(e) $\frac{2\pi^2}{3}$

15. The area of the region bounded by the graph of $f(x) = 2^x - 2$ and the x-axis from $x = 0$ to $x = 2$ is equal to

(a) $\dfrac{5}{\ln 2}$

(b) $\dfrac{1}{\ln 2}$

(c) $\dfrac{2}{\ln 2}$

(d) $\dfrac{4}{\ln 2}$

(e) $\dfrac{3}{\ln 2}$

(082)

1. The average value of the function $f(x) = \cos^4 x \sin x$ over $[0, \pi]$ is

(a) $\dfrac{2}{5\pi}$

(b) $\dfrac{2}{\pi}$

(c) $\dfrac{2}{5}$

(d) $\dfrac{5\pi}{2}$

(e) $\dfrac{2}{3}$

9. The area of the region enclosed by the graphs of $y = x - 1$ and $x = (y - 1)^2$ is equal to

(a) $\dfrac{9}{2}$

(b) 9

(c) 3

(d) $\dfrac{8}{3}$

(e) 8

13. The volume of the solid obtained by rotating the region enclosed by the curves $y = \cosh x$, $y = \sinh x$, $x = 0$ and $x = 5$, about the x-axis, is (Hint: $\cosh x - \sinh x > 0$)

(a) 5π

(b) $\dfrac{\pi}{5}$

(c) $\dfrac{5\pi}{2}$

(d) π

(e) $\dfrac{\pi}{5} - 1$

25. The volume of the solid obtained by rotating the region enclosed by $y = \dfrac{1}{x^2 + 2x + 2}$, $x = 1$, $x = 2$, about the line $x = -1$ is

(a) $\pi \ln 2$

(b) $2\pi \ln 2$

(c) $\pi \ln 2 - 2\pi(\tan^{-1} 3 - \tan^{-1} 2)$

(d) $2\pi(\tan^{-1} 3 - \tan^{-1} 2)$

(e) $\pi(\tan^{-1} 2 - \tan^{-1} 3)$

(081)

9. The area of the region bounded by the parabolas $y = (x+1)^2$ and $y^2 = x + 1$ is equal to

(a) $\frac{2}{3}$

(b) $\frac{4}{3}$

(c) $\frac{1}{3}$

(d) 1

(e) $\frac{5}{3}$

12. If the region enclosed by the curves $y = \sin(x^2)$, $y = 0$, $x = 0$, and $x = \sqrt{\pi}$ is rotated about the y-axis, then the volume of the generated solid is

(a) $2\pi + 1$

(b) $\dfrac{\pi}{3}$

(c) 4π

(d) $\pi - 2$

(e) 2π

14. Let R be the region in the first quadrant that is bounded by the curves $y = \sqrt[3]{x}$ and $y = x^3$. The volume of the solid obtained by rotating R about the line $y = 1$ is given by

(a) $\pi \int_0^1 [(1 - x^3)^2 - (1 - \sqrt[3]{x})^2]\, dx$

(b) $\pi \int_0^1 (y^{2/3} - y^2)\, dy$

(c) $2\pi \int_0^1 (1 - x)(\sqrt[3]{x} - x^3)\, dx$

(d) $\pi \int_0^1 (x^3 - \sqrt[3]{x})^2\, dx$

(e) $\pi \int_0^1 [(\sqrt[3]{y} - 1)^2 - (y^3 - 1)^2]\, dy$

(073)

1. The area of the region enclosed by the curves $y = 2x^2 - 1$ and $y = -2x - 1$ is equal to

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{4}$

(e) $\frac{2}{5}$

2. If the region enclosed by the curves $y = \sin x$ and $y = 0$ between $x = 0$ and $x = \pi$ is revolved about the $y-axis$, then the volume of the solid generated is equal to

(a) $2\pi^2$

(b) π^2

(c) $\dfrac{\pi^2}{2}$

(d) $\dfrac{\pi^2}{4}$

(e) $4\pi^2$

13. If the region enclosed by the curves $y = e^x$, $x = 0$, $x = \ln 2$ and $y = 0$ is revolved about the line $y = -1$, then the volume of the solid generated is equal to

(a) $\dfrac{7\pi}{2}$

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{9\pi}{2}$

(d) π

(e) $\dfrac{5\pi}{2}$

(072)

1. The volume of the solid generated by revolving the region enclosed by the curve $y = \sin(x^2)$ and the x-axis over the interval $[0, \sqrt{\pi}]$ about the y-axis is

(a) π

(b) 2π

(c) -2π

(d) $\frac{\pi}{2}$

(e) $3\sqrt{\pi}$

2. The volume obtained by rotating the region bounded by $y = x^4$, $y = 1$ about $y = 1$ equals

(a) $\frac{2}{9}\pi$

(b) $\frac{22}{45}\pi$

(c) $\frac{64}{45}\pi$

(d) $\frac{34}{45}\pi$

(e) $\frac{44}{45}\pi$

3. The area of the region enclosed by the graphs of $y^2 = x$ and $y = x - 2$ is equal to

(a) $\int_{-1}^{2}(2 + y - y^2)dy$

(b) $\int_{1}^{4}(x^2 - x + 2)dx$

(c) $\int_{1}^{4}(\sqrt{x} - x + 2)dx$

(d) $\int_{-1}^{2}(x^2 - x + 2)dx$

(e) $\int_{1}^{4}(2 + y - y^2)dy$

(071)

3. The area of the region bounded by the graphs of $y = x^2 - 2$ and $y = x$ is

(a) $\frac{9}{2}$

(b) $\frac{3}{2}$

(c) $\frac{7}{2}$

(d) $\frac{5}{2}$

(e) $\frac{11}{2}$

5. The volume of the solid generated by rotating the region enclosed by the curves $y = x$ and $y = \sqrt{x}$ about the y-axis is

(a) $\pi \int_0^1 (y^2 - y^4) dy$

(b) $\pi \int_0^1 (y - y^2) dy$

(c) $\pi \int_0^1 (x^2 - x)\, dx$

(d) $\pi \int_{-1}^1 (y + y^2) dy$

(e) $\pi \int_{-1}^0 (x - x^2)\, dx$

11. The average value of the function $f(x) = \dfrac{x}{(x+3)^3}$ over the interval $[-1, 1]$ is

(a) $\dfrac{-1}{64}$

(b) $\dfrac{3}{32}$

(c) $\dfrac{-5}{32}$

(d) $\dfrac{5}{64}$

(e) 0

22. The area of the region between the x-axis and the curve $y = \dfrac{x}{e^x}$ for $x \geq 0$ is

(a) 1

(b) 2

(c) $\dfrac{1}{2}$

(d) $\dfrac{3}{2}$

(e) 3

27. If the region bounded by the curves $y = \sqrt{x-1}$, $y = 0$, and $x = 5$ is rotated about the line $y = 3$, then the volume of the generated solid is

(a) 24π

(b) 10π

(c) 6π

(d) 36π

(e) 4π

(063)

5. Let R be the region between the graphs of $f(x) = 5x$ and $g(x) = x^2$ on $[0, 3]$. If R is revolved about the x-axis then the volume of the solid obtained equals

(a) $\dfrac{5}{882\pi}$

(b) $\dfrac{5\pi}{882}$

(c) $\dfrac{882}{5}$

(d) 882π

(e) $\dfrac{882\pi}{5}$

11. The average value of $f(x) = \sin^2(x)$ over $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is equal to

(a) $-\frac{\pi}{4}$

(b) $\frac{\pi}{4}$

(c) $-\frac{1}{2}$

(d) $\frac{\pi - 2}{2\pi}$

(e) $\frac{\pi + 2}{2\pi}$

12. The area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$ is equal to

(a) 9

(b) 18

(c) $\frac{9}{2}$

(d) 27

(e) $\frac{27}{2}$

1. The region bounded by the curves $y = 4x - x^2$ and $y = 8x - 2x^2$ is rotated about the line $x = -4$. Then the volume of the resulting solid is given by

(a) $2\pi \int_0^4 (x+4)(x^2 - 4x)dx$

(b) $2\pi \int_{-4}^4 (x+4)(x^2 - 4x)dx$

(c) $2\pi \int_0^4 (x-4)(x^2 - 4x)dx$

(d) $2\pi \int_0^4 (x-4)(4x - x^2)dx$

(e) $2\pi \int_0^4 (x+4)(4x - x^2)dx$

5. The area between the curves $y = \sin 2x$ and $y = \cos x$ from $x = 0$ to $x = \dfrac{\pi}{2}$ is equal to

(a) 0

(b) 1

(c) $\dfrac{1}{2}$

(d) $\dfrac{1}{4}$

(e) $\sqrt{3} - \dfrac{3}{2}$

11. The average value of $f(x) = \sqrt{9 - x^2}$ on $[0, 3]$ is equal to

(a) $\dfrac{10\pi}{4}$

(b) $\dfrac{3\pi}{16}$

(c) $\dfrac{3\pi}{4}$

(d) $\dfrac{9\pi}{16}$

(e) 3π

(061)

17. The area of the region enclosed by the graphs of $y = \frac{1}{x}$, $y = \frac{1}{x^2}$ and $x = 2$ is equal to

(a) $\ln 2 + \frac{3}{2}$
(b) $\frac{1}{2} - \ln 2$
(c) $\ln 2 + \frac{1}{2}$
(d) $\ln \frac{1}{2} - \frac{1}{2}$
(e) $\ln 2 - \frac{1}{2}$

18. The volume of the solid obtained by rotating the region bounded by the curves of $y = \sqrt{x-1}$, $x = 2$, $x = 5$ and $y = 0$ about the x-axis is equal to

(a) 21π
(b) 15π
(c) $15\pi/2$
(d) $21\pi/2$
(e) $15\pi/4$

19. The volume generated by rotating the region bounded by the graphs of $y = x^2$, $y = 0$, $x = 1$, and $x = 2$ about $x = 4$ is equal to

(a) $14\pi/3$
(b) $67\pi/2$
(c) 67π
(d) $67\pi/6$
(e) $7\pi/6$

Answer Key :

Question	Answer
8 (092)	A
9 (092)	A
10 (092)	A
24(092)	A
1 (083)	D
6(083)	D
13 (083)	C
15(083)	B
1(082)	A
9 (082)	A
13 (082)	A
25 (082)	A
9(081)	C
12 (081)	E
14 (081)	A
1 (073)	A
2 (073)	A
13(073)	A
1(072)	B
2(072)	C
3 (072)	A
3 (071)	A
5 (071)	A
11 (071)	A

22 (071)	A
27 (071)	A
5(063)	--
11 (063)	--
12(063)	--
1 (062)	E
5(062)	C
11(062)	C
17(061)	E
18(061)	C
19(061)	D

Old Exam 7.1 :

(092)

Q.2. $(5-\ \mathit{points})$. Evaluate $\int \ln(2x)\,dx$.

(091)

(a) [**9 points**] $\displaystyle\int x(\ln x)^2\,dx$.

(082)

(a) (5 points) . $\int x\tan^{-1}x\,dx$

(071)

(b) $\displaystyle\int e^{-x}\cos(2x)\,dx$.

(d) $\displaystyle\int x^3\,e^{x^2}\,dx$.

(061)

11. The integral $\int_1^e \sin(\ln x)dx$ is equal to

(a) $\frac{1}{2}(1 + e\sin 1 - e\cos 1)$
(b) $\frac{1}{2}(1 + \sin 1 - \cos 1)$
(c) $1 + e\sin 1 - e\cos 1$
(d) $\frac{1}{2}(e\sin 1 - e\cos 1)$
(e) $\frac{1}{2}(1 + e\sin 1 + e\cos 1)$

Old Exam 7.2 :

(092)

Q.5. $(8-\ points)$. Evaluate $\displaystyle\int \frac{\tan^5 x}{\sqrt[3]{\sec x}} dx.$

Q.7. $(9-\ points)$. Evaluate $\displaystyle\int \csc^3 x\, dx.$

Q.10. $(9-\ points)$. $\displaystyle\int_0^{\pi/3} \sin^3\theta \sec^2\theta\, d\theta.$

(083)

(a) [**7 points**] $\displaystyle\int \frac{\sec^6\theta}{\tan^2\theta}\, d\theta.$

(082)

Q.5.(b) $(5\ points)$. $\int \tan^2 x \sec^4 x\, dx.$

(081)

(a) **[6 points]** $\int \sin^5(3t)\cos^4(3t)\,dt.$

(072)

(a) (5 points) $\int \sin^2 3x\,dx.$

(b) (5 points) $\int \sin^{\frac{3}{2}} x\ \cos^3 x\,dx.$

(071)

(a) $\int \tan^2 t \sec^4 t\,dt.$

(063)

1. $\displaystyle\int_0^{\pi/2} \cos^3(x)dx =$

(a) $\dfrac{1}{4}$

(b) 0

(c) $\dfrac{2}{3}$

(d) $-\dfrac{2}{3}$

(e) $-\dfrac{1}{4}$

2. $\displaystyle\int \cos(3x)\cos(2x)\ dx =$

(a) $\sin(3x)\sin(2x) + c$

(b) $\dfrac{1}{6}\sin(3x)\sin(2x) + c$

(c) $\dfrac{\sin(5x)}{5} + c$

(d) $\dfrac{1}{2}\sin(x) + \dfrac{1}{10}\sin(5x) + c$

(e) $\dfrac{1}{3}\sin(3x) + \dfrac{1}{4}\sin(2x) + c$

4. $\displaystyle\int \tan^6(x)dx =$

(a) $\dfrac{\tan^5(x)}{5} - \dfrac{\tan^3(x)}{3} + \tan(x) - x + c$

(b) $\dfrac{\tan^7(x)}{7} + c$

(c) $\dfrac{\tan^5(x)}{5} + \dfrac{\tan^3(x)}{3} + \tan(x) + x + c$

(d) $\ln|\sec^6(x)| + c$

(e) $\sec^6(x) + c$

(061)

7. The integral $\displaystyle\int_0^{\pi/2} \sin^6 x \ \cos^3 x \, dx$ is equal to

(a) 18
(b) 1/28
(c) 2/63
(d) 1/63
(e) 3/28

9. The integral $\int \tan^2 x \ \ \sec x \, dx$ is equal to

(a) $\frac{1}{3}\tan^3 x + C$
(b) $\frac{1}{2}\sec^2 x \ \tan^3 x - \ln\left|\sec^2 x + \tan^3 x\right| + C$
(c) $\frac{1}{3}\sec^3 x + C$
(d) $\frac{1}{2}(\sec x \ \tan x - \ln|\sec x + \tan x|) + C$
(e) $\frac{1}{2}(\sec^2 x \ \tan^3 x + \ln\left|\sec^2 x + \tan^3 x\right|) + C$

Old Exam 7.3 :

(092)

Q.6. (7 $-$ *points*). Evaluate $\int \frac{dx}{(16-x^2)^{3/2}}$.

(091)

(b) **[10 points]** $\int \frac{x^3}{\sqrt{4-x^2}}\,dx.$

(083)

(b) **[10 points]** $\int \sqrt{5+4x-x^2}\,dx.$

(082)

Q.5.(c). (8 points). $\int \frac{(x-3)\,dx}{\sqrt{8x-x^2}}$.

(081)

(d) **[7 points]** $\displaystyle\int_{2/5}^{4/5} \frac{\sqrt{25x^2-4}}{x}\,dx.$

(072)

4. (6 points) Evaluate $\displaystyle\int \frac{\sqrt{x^2-4x}}{x-2}\,dx.$

(071)

(c) $\displaystyle\int \frac{1}{x^3\sqrt{x^2-1}}\,dx.$

(f) $\displaystyle\int \frac{1}{(-x^2-2x)^{3/2}}\,dx.$

(061)

15. The integral $\int x\sqrt{1-x^4}dx$ is equal to

(a) $\frac{-2x^5}{\sqrt{1-x^4}}\sin^{-1}(x)+\frac{1}{4}x^2\sqrt{1-x^4}+C$
(b) $\frac{1}{2}\sin^{-1}(x)+\frac{1}{4}x\sqrt{1-x^4}+C$
(c) $\frac{1}{4}x^2+\frac{1}{8}x^2\sin 2x^2+C$
(d) $\frac{1}{2}\tan^{-1}(x^2)+\frac{3}{4}x\sqrt{1-x^4}+C$
(e) $\frac{1}{4}\sin^{-1}(x^2)+\frac{1}{4}x^2\sqrt{1-x^4}+C$

Old Exam 7.4 :

(092)

Q.1. $(7 - \ points)$. Write out the form of partial fraction decomposition of the expression

$$\frac{x^2}{(x+1)(x-2)^3(x^2+1)(x^2+x+1)^2}.$$

[Do not determine the numerical value of the coefficients]

Q.8. $(12 - \ points)$. Evaluate $\displaystyle\int \frac{4x^2+13x+15}{x^3+4x^2+5x}dx.$

Q.11. $(8 - \ points)$. Evaluate $\displaystyle\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}.$

Q.13. $(8 - \ points)$. Evaluate $\displaystyle\int \frac{1}{1+\sin x+\cos x}dx.$

[You may use the substitution $t = \tan\dfrac{x}{2}$]

(091)

(c) [12 points] $\displaystyle\int \frac{x^3+1}{x^3+x}\,dx.$

(d) [**10 points**] $\displaystyle\int \frac{\sec x}{2+\tan x}\,dx.$ Hint: Use the substitution $t = \tan\left(\frac{x}{2}\right)$.

(083)

(c) [**10 points**] $\displaystyle\int \frac{x^4+3x^3+2x^2+1}{(x^2+x)(x+2)}\,dx.$

(d) [**8 points**] $\displaystyle\int \frac{1}{1+\cos x-\sin x}\,dx.$

(082)

Q.5.(d) . (8 points) . $\int \dfrac{dx}{3-5\sin x}$

Q.5.(e) (9 points) . $\int \dfrac{x^2+7x+11}{(x^2+6x+13)(x-2)}dx.$

Q.5. (f) (6 points) . $\int \frac{1}{x+\sqrt[3]{x}}dx.$

(081)

(b) **[6 points]** $\displaystyle\int \frac{\sqrt{x}}{x+4}\, dx.$

(c) **[8 points]** $\displaystyle\int \frac{1}{1+2\sin x}\, dx.$ Hint: Use the substitution $t = \tan\left(\frac{x}{2}\right)$.

(e) **[9 points]** $\displaystyle\int \frac{4x}{(x+1)(x-1)^2}\, dx.$

(072)

1. (4 points) Write out the form of partial fraction decomposition of $\dfrac{x+2}{(x-2)^2(x^2+4)^2}$.
   Do not determine the numerical values of the coefficients.

(c) (7 points) $\displaystyle\int \frac{dx}{1-\sin x+\cos x}$ [Hint: you may use the substitution $t = \tan\frac{x}{2}$]

6. (8 points) Evaluate $I = \displaystyle\int \frac{x^2 + 2x - 1}{x^3 - x} dx$

(071)

(e) $\displaystyle\int \frac{1}{5 + 3\cos x}\, dx$

4. (a) Write out the form of the partial fraction decomposition of $\dfrac{x-2}{x(x^3+x)^2}$. Do not determine the numerical values of the coefficients. (5 points)

(b) Evaluate $\displaystyle\int \frac{x^3 - 4x - 10}{x^2 - x - 6}\, dx.$

(063)

Q2. **(25 points)** Find $\displaystyle\int \frac{1}{x^4 - x^3 - x + 1}\, dx.$

6. $\displaystyle\int_0^{\pi/2} \frac{1}{1+\sin(x)+\cos(x)}\,dx =$

(a) 0

(b) 2

(c) $2\ln 2$

(d) $\ln 2$

(e) $-\ln 2$

10. $\displaystyle\int \frac{dx}{x^2+2x+10}$ equals

(a) $\displaystyle\frac{1}{3}\arcsin\frac{x+1}{3}+c$

(b) $\arctan(x+1)+c$

(c) $\displaystyle\frac{1}{3}\arctan\frac{x+1}{3}+c$

(d) $\displaystyle\arctan\frac{x+1}{3}+c$

(e) $\displaystyle\frac{1}{3}\arctan(x+1)+c$

(061)

6. The integral $\int \frac{dx}{x^3+x}$ is equal to

(a) $\ln|x| - \frac{1}{2}(x^2+1) + C$
(b) $\ln\left|x^3+x\right| + C$
(c) $-\frac{1}{4}x^{-4} - \frac{1}{2}x^2 + \tan^{-1}x^2 + C$
(d) $\ln|x| - \frac{1}{2}\ln(x^2+1) + C$
(e) $\ln|x| + \frac{1}{2}\ln(x^2+1) + \frac{1}{(x^3+x)^2} + C$

10. The integral $\int \sqrt{\frac{1-x}{1+x}}dx$ is equal to

(a) $\tan^{-1}x + \sqrt{1-x^2} + C$
(b) $\ln\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}\sqrt{1-x^2} + C$
(c) $\ln\sqrt{1-x^2} - \sin^{-1}\sqrt{1-x^2} + C$
(d) $\ln\left|1-x^2\right| + \sin^{-1}x + C$
(e) $\sin^{-1}x + \sqrt{1-x^2} + C$

12. The integral $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}}$ is equal to

(a) $2\sqrt{x}+4\sqrt[4]{x}+4\ln(\sqrt[4]{x}+1)\sin^{-1}\sqrt[4]{x}+C$
(b) $2\sqrt{x}+4\ln(\sqrt[4]{x}+1)+\frac{1}{(\sqrt{x}+\sqrt[4]{x})^2}+C$
(c) $2\sqrt{x}-4\sqrt[4]{x}+4\ln(\sqrt[4]{x}+1)+C$
(d) $2x^2-4x+4\ln|x+1|-\tan^{-1}\sqrt[4]{x}+C$
(e) $\ln|\sqrt{x}+\sqrt[4]{x}|+C$

Old Exam 7.8 :

(092)

Q.12. $(7-\ \mathit{points})$. Determine whether the integral $\displaystyle\int_0^1 \frac{dx}{(x-1)^{2/3}}$ converges or diverges.

(091)

3. Determine whether the integral is convergent or divergent. If it is convergent, find its value.

   (a) [**6 points**] $\displaystyle\int_0^9 \frac{1}{x\sqrt{x}}\,dx$.

   (b) [**8 points**] $\displaystyle\int_0^{+\infty} x\, e^{-10x}\, dx$.

(083)

2. [**8 points**] Evaluate the integral or show that it is divergent:

$$\int_0^2 \frac{1}{(1-2x)^{4/3}}\,dx.$$

(082)

Q.4. (b). (4 points). $\int_2^{+\infty} \frac{1}{x \ln x} dx$

(081)

4. (a) Determine if the integral is improper or not. Justify.

(i) **[3 points]** $\displaystyle\int_2^5 \ln(x-1)\, dx.$

(ii) **[3 points]** $\displaystyle\int_1^5 \frac{1}{x^2-x}\, dx.$

(072)

7. (7 points) Compute $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\, dx.$

(071)

5. Evaluate the integral or show that it is divergent.

(a) $\displaystyle\int_{-\infty}^0 \frac{3x}{(5x^2+6)^2}\, dx.$

(b) $\displaystyle\int_0^2 \frac{1}{(x-1)^3}\,dx.$

(063)

3. The improper integral $\displaystyle\int_e^{\infty} \frac{1}{x\ln(x)}\,dx$

(a) converges to 1

(b) converges to 2

(c) converges to π

(d) converges to $\frac{1}{2}$

(e) diverges

7. $\displaystyle\int_0^3 \frac{dx}{(x-1)^2} =$

(a) $-\dfrac{3}{2}$

(b) $\dfrac{3}{2}$

(c) does not exist

(d) $-\dfrac{2}{3}$

(e) 0

(061)

13. The integral $\displaystyle\int_{-\infty}^{0} xe^{-x^2}\,dx$

(a) converges to 1
(b) converges to $-1/2$
(c) converges to -1
(d) diverges
(e) converges to 0

Review Chapter 7 .

Review Chapter 7 :

(092)

2. $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} x^3 \sin(x^2)dx =$

(a) $\dfrac{\pi - 1}{2}$

(b) $\dfrac{\pi + 1}{4}$

(c) $\dfrac{\pi}{2}$

(d) $\pi + 1$

(e) $\pi - 1$

3. $\int \cot^3 \alpha \csc^3 \alpha \, d\alpha =$

(a) $\frac{1}{3} \csc^3 \alpha - \frac{1}{5} \csc^5 \alpha + C$

(b) $\frac{1}{3} \csc^3 \alpha + \frac{1}{5} \csc^5 \alpha + C$

(c) $\frac{1}{2} \csc^2 \alpha + \frac{1}{4} \csc^4 \alpha + C$

(d) $\frac{1}{2} \csc^2 \alpha - \frac{1}{4} \csc^4 \alpha + C$

(e) $\frac{1}{4} \csc^4 \alpha + C$

4. The improper integral $\int_{-\infty}^{1} \frac{1}{2} e^{2x}\, dx$ is

(a) convergent and its value is $e^2/4$

(b) convergent and its value is $e^3/8$

(c) convergent and its value is $e/2$

(d) convergent and its value is e

(e) divergent

11. $\displaystyle\int \frac{dx}{x(x^2+1)} =$

(a) $\ln\left(\dfrac{|x|}{\sqrt{x^2+1}}\right) + C$

(b) $\ln\left(|x|\sqrt{x^2+1}\right) + C$

(c) $\ln\left(\dfrac{\sqrt{x^2+1}}{|x|}\right) + C$

(d) $\ln(|x|(x^2+1)) + C$

(e) $\ln\left(\dfrac{x^2+1}{|x|}\right) + C$

25. $\displaystyle\int \frac{x}{\sqrt{3-2x-x^2}}\, dx =$

(a) $-\sqrt{3-2x-x^2} - \sin^{-1}\left(\dfrac{x+1}{2}\right) + C$

(b) $\sqrt{3-2x-x^2} + \cos^{-1}\left(\dfrac{x+1}{2}\right) + C$

(c) $-\sqrt{3-2x-x^2} + \sin\left(\dfrac{x+1}{2}\right) + C$

(d) $\sqrt{3-2x+x^2} + \cos^{-1}\left(\dfrac{x+1}{2}\right) + C$

(e) $\sqrt{3-2x-x^2} + C$

26. The improper integral $\int_0^3 \frac{3\,dx}{x^2 - 5x + 4}$ is

(a) divergent

(b) convergent and its value is $\ln 4$

(c) convergent and its value is $\ln 3$

(d) convergent and its value is $\ln 2$

(e) convergent and its value is 0

1. $$\int \frac{(2-\sqrt{x})^2}{x}\,dx =$$

(a) $x - 8\sqrt{x} + 4\ln|x| + C$

(b) $x - 2\sqrt{x} + 4\ln|x| + C$

(c) $x - \dfrac{8}{\sqrt{x}} + 4\ln|x| + C$

(d) $1 - 4\sqrt{x} + \ln|x| + C$

(e) $2x - \dfrac{1}{4}\sqrt{x} + 2\ln|x| + C$

2. $$\int \frac{1}{x\sqrt{25-(\ln x)^2}}\,dx =$$

(a) $\sin^{-1}\left(\dfrac{\ln x}{5}\right) + C$

(b) $\sin^{-1}\left(\dfrac{\ln x}{\sqrt{5}}\right) + C$

(c) $\dfrac{1}{5}\sin^{-1}(\ln x) + C$

(d) $\sin^{-1}\left(\cos\left(\dfrac{x}{5}\right)\right) + C$

(e) $\dfrac{1}{5}\sin^{-1}\left(\dfrac{\ln x}{5}\right) + C$

6. $\displaystyle\int_0^{\pi/4} \tan^4 x \; dx =$

(a) $\dfrac{\pi}{4} - \dfrac{2}{3}$

(b) $\dfrac{\pi}{3} - \dfrac{1}{2}$

(c) $\dfrac{\pi}{2} + \dfrac{1}{2}$

(d) $\dfrac{\pi}{4} - \dfrac{1}{2}$

(e) $\dfrac{\pi}{3} + \dfrac{2}{3}$

8. The improper integral $\int_{-\infty}^{1} \frac{x}{(1+x^2)^3}\, dx$ is

(a) convergent and its value is $\frac{-1}{16}$

(b) convergent and its value is $\frac{2}{9}$

(c) convergent and its value is $\frac{-3}{8}$

(d) convergent and its value is $\frac{3}{16}$

(e) divergent

12. If $\frac{8x^2+7x-6}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$, then $A+B+C=$

(a) 6

(b) -5

(c) 3

(d) 0

(e) -1

18. $\displaystyle\int_2^4 \frac{\sqrt{x^2-4}}{x}\,dx =$

(a) $2\sqrt{3} - \dfrac{2\pi}{3}$

(b) $1 - \dfrac{2\pi}{3}$

(c) $\sqrt{3} - \pi$

(d) $2\sqrt{3} + \dfrac{\pi}{3}$

(e) $\sqrt{3} - \dfrac{\pi}{2}$

22. $\displaystyle\int \frac{\sqrt{x}}{x+4}\,dx =$

(a) $2\sqrt{x} - 4\tan^{-1}\left(\dfrac{\sqrt{x}}{2}\right) + C$

(b) $2\sqrt{x} + 2\tan^{-1}\left(\dfrac{\sqrt{x}}{2}\right) + C$

(c) $\sqrt{x} - 2\tan^{-1}(\sqrt{x}) + C$

(d) $\sqrt{x} - 4\tan^{-1}\left(\dfrac{\sqrt{x}}{2}\right) + C$

(e) $4\sqrt{x} + 2\tan^{-1}(\sqrt{x}) + C$

25. $\displaystyle\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx =$

(a) $x - \sqrt{1-x^2}\cdot\sin^{-1}x + C$

(b) $\ln|1-x| - \sqrt{1-x^2}\cdot\sin^{-1}x + C$

(c) $\sqrt{1-x^2} - \sin^{-1}x + C$

(d) $\sqrt{1-x^2}(1-\sin^{-1}x) + C$

(e) $\dfrac{1}{2}(\sin^{-1}x)^2 + C$

(083)

2. $\displaystyle\int \sin^2 x \cos^3 x \, dx =$

(a) $\dfrac{1}{12}\sin^3 x \cos^4 x + C$

(b) $\dfrac{1}{6}\sin^3 x - \dfrac{1}{10}\cos^5 x + C$

(c) $\dfrac{1}{3}\cos^3 x - \dfrac{1}{5}\cos^5 x + C$

(d) $\sin^2 x - \sin^4 x + C$

(e) $\dfrac{1}{3}\sin^3 x - \dfrac{1}{5}\sin^5 x + C$

9. $\displaystyle\int \frac{2x^2}{\sqrt[3]{1+x^3}}\,dx =$

(a) $2(1+x^3)^{1/3} + C$

(b) $(1+x^3)^{2/3} + C$

(c) $-\frac{1}{4}(1+x^3)^{-4/3} + C$

(d) $\frac{2}{3}(1+x^3)^{-2/3} + C$

(e) $2\ln|\sqrt[3]{1+x^3}| + C$

11. If $\displaystyle\frac{3x^2+2x+1}{(x-1)(x^2+2x+5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+5}$, then $A+B-C=$

(a) $-\frac{3}{4}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

(d) $-\frac{1}{2}$

(e) 0

14. $\displaystyle\int_0^{\pi/2} e^{\sin t}\sin(2t)\,dt =$

(a) 2

(b) $-\dfrac{1}{2}$

(c) 0

(d) -3

(e) 4

27. $$\int \frac{1}{x\sqrt{x-4}}\,dx =$$

(a) $\ln|x| + \ln(\sqrt{x-4}) + C$

(b) $\dfrac{1}{2}\ln\left|\dfrac{\sqrt{x-4}-2}{\sqrt{x-4}+2}\right| + C$

(c) $\tan^{-1}\left(\dfrac{\sqrt{x-4}}{2}\right) + C$

(d) $\dfrac{\sqrt{x-4}}{x} + C$

(e) $2\tan^{-1}\left(\dfrac{\sqrt{x-4}}{2}\right) + C$

28. The improper integral $\int_0^{+\infty} \frac{e^{-1/x}}{x^2}\, dx$ is

(a) convergent and its value is 0

(b) convergent and its value is 1

(c) convergent and its value is e^{-1}

(d) convergent and its value is $1 - e$

(e) divergent

(082)

2. $\int \tan^4 x \, dx =$

(a) $\frac{1}{3}\tan^3 x - \tan x + x + c$

(b) $\frac{1}{3}\tan^3 x + \tan x \sec x + c$

(c) $\ln|\sec x + \tan x| + \tan^3 x + c$

(d) $\ln|\csc x - \sec x| + \tan x + c$

(e) $\sec^3 x + 3\sec^2 x + c$

5. $\displaystyle\int \frac{1}{x^2\sqrt{25-x^2}}\,dx =$

(a) $\dfrac{-1}{25}\dfrac{\sqrt{25-x^2}}{x}+c$

(b) $\dfrac{1}{125}\dfrac{\sqrt{25-x^2}}{x}+c$

(c) $\dfrac{-1}{5}\dfrac{\sqrt{25-x^2}}{x}+c$

(d) $\dfrac{1}{5}\dfrac{\sqrt{25-x^2}}{x}+c$

(e) $\dfrac{\sqrt{25-x^2}}{x}+c$

7. $\displaystyle\int \frac{x^2 - x + 6}{x^3 + x}\, dx =$

(a) $6\ln|x| - \dfrac{5}{2}\ln(x^2+1) - \tan^{-1}(x) + c$

(b) $6\ln|x| - \dfrac{5}{2}\ln(x^2+1) + \tan^{-1}(x) + c$

(c) $6\ln|x| - \dfrac{5}{2}\ln(x^2+1) - \sin^{-1}(x) + c$

(d) $\ln|x| - \dfrac{1}{2}\ln(x^2+1) - \tan^{-1}(x) + c$

(e) $-\dfrac{5}{2}\ln(x^2+1) - \tan^{-1}(x) + c$

8. $\displaystyle\int_1^3 \frac{\sqrt{x}}{x^2 + x} dx =$

(a) $\dfrac{\pi}{6}$

(b) $\dfrac{\pi}{3}$

(c) 2π

(d) 5π

(e) $\dfrac{5\pi}{4}$

10. The improper integral $\int_0^{\pi/2} \frac{\cos x}{1 - \sin x}\, dx$

(a) diverges

(b) converges and has the value 0

(c) converges and has the value $\frac{\pi}{4}$

(d) converges and has the value π

(e) converges and has the value $\frac{\pi}{2}$

26. $\displaystyle\int_0^{\pi^2/4} \cos\sqrt{x}\, dx =$

(a) $\pi - 2$

(b) $\dfrac{\pi}{2} - 1$

(c) $\dfrac{\pi}{2} - \dfrac{1}{2}$

(d) $\dfrac{\pi^2}{4} - 1$

(e) $\dfrac{\pi}{4} - 1$

(081)

2. $\int \left(1 - \frac{1}{x}\right)^2 \, dx =$

(a) $x - 2\ln|x| - \dfrac{2}{x^3} + C$

(b) $x - \dfrac{1}{x} - 2\ln|x| + C$

(c) $\dfrac{1}{3}\left(1 - \dfrac{1}{x}\right)^3 + C$

(d) $1 - \dfrac{1}{x} + C$

(e) $x + \dfrac{1}{x} - 2\ln|x| + C$

3. $\displaystyle\int_0^{\pi/2} \frac{\cos t}{1 + \sin^2 t} \, dt =$

(a) 1

(b) $\ln 2$

(c) $\dfrac{\pi}{4}$

(d) 0

(e) $\dfrac{\pi}{3}$

6. $\displaystyle\int \tan^3(2x)\sec^5(2x)\,dx =$

(a) $\dfrac{\tan^7(2x)}{14} - \dfrac{\tan^5(2x)}{10} + C$

(b) $\dfrac{\sec^7(2x)}{7} - \dfrac{\sec^5(2x)}{5} + C$

(c) $\dfrac{\sec^7(2x)}{14} - \dfrac{\sec^5(2x)}{10} + C$

(d) $\dfrac{\tan^4(2x)}{4} - \dfrac{\sec^6(2x)}{6} + C$

(e) $\sec^6(2x) - \sec^4(2x) + C$

7. $\displaystyle\int_{e}^{e^7} \frac{dx}{x\sqrt{2+2\ln x}}\, dx =$

(a) 10

(b) 2

(c) 6

(d) 4

(e) 8

8. $\displaystyle\int x \ln x \, dx =$

(a) $\dfrac{1}{2}\, x^2 \ln x - \dfrac{1}{2}\, x^2 + C$

(b) $\dfrac{1}{2}\, x^2 \ln x - \dfrac{1}{4}\, x^2 + C$

(c) $x \ln x + \dfrac{1}{2}\, x^2 + C$

(d) $\dfrac{1}{2}\, x^2 \ln x + \dfrac{1}{2}\, x(\ln x)^2 + C$

(e) $\dfrac{1}{2}\, (\ln x)^2 + C$

10. The form of the partial fraction decomposition of $\dfrac{x^3+1}{x^2(x^2+4)^2}$ is

(a) $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{(x+2)^2}+\dfrac{D}{(x-2)^2}$

(b) $\dfrac{A}{x^2}+\dfrac{B}{x^2+4}+\dfrac{Cx+D}{(x^2+4)^2}$

(c) $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{Cx+D}{x^2+4}+\dfrac{Ex+F}{(x^2+4)^2}$

(d) $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^2+4}+\dfrac{D}{(x^2+4)^2}$

(e) $\dfrac{Ax+B}{x^2}+\dfrac{Cx+D}{(x^2+4)^2}$

13. $\int \dfrac{x^4 + x^2 - 1}{x^3 + x}\, dx =$

(a) $\frac{1}{2}\, x^2 - \ln|x| + \frac{1}{2} \ln\ |x^3 + x| + C$

(b) $\frac{1}{2}\, x^2 - \ln|x| + \frac{1}{2} \ln\ |x^2 + 1| + C$

(c) $\frac{1}{2}\, x^2 - \ln|x| + C$

(d) $\ln|x^4 + x^2 - 1| - \frac{1}{2} \ln|x^3 + x| + C$

(e) $\frac{1}{2}\, x^2 + \ln|x| - \frac{1}{x} + C$

16. The improper integral $\int_0^2 \dfrac{1}{\sqrt[5]{x-1}}\, dx$

(a) converges and its value is $\frac{1}{4}$

(b) converges and its value is $\frac{5}{4}$

(c) converges and its value is $\frac{5}{2}$

(d) converges and its value is 0

(e) diverges

18. $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\,dx =$

(a) $8\sin^{-1}\left(\frac{x}{4}\right) - \frac{1}{2}\,x\sqrt{16-x^2} + C$

(b) $8\sin^{-1}\left(\frac{x}{4}\right) - x + C$

(c) $\sqrt{16-x^2} + C$

(d) $8\sin^{-1}\left(\frac{x}{4}\right) - \sqrt{16-x^2} + C$

(e) $4\sin^{-1}\left(\frac{x}{4}\right) + 2x\sqrt{16-x^2} + C$

(073)

5. $\displaystyle\int \frac{\sqrt{1-x^2}-x}{\sqrt{1-x^2}}\ dx =$

(a) $x+\sqrt{1-x^2}+c$

(b) $x-\sin^{-1}x+c$

(c) $x-\frac{1}{2}\sqrt{1-x^2}+c$

(d) $x+\sin^{-1}x+c$

(e) $x+\frac{3}{2}\sqrt{1-x^2}+c$

9. $\displaystyle\int \tan^{-1}\left(\frac{1}{x}\right)\ dx =$

(a) $x\tan^{-1}\left(\frac{1}{x}\right)+\ln\sqrt{1+x^2}+c$

(b) $\frac{1}{2}\ x\tan^{-1}\left(\frac{1}{x}\right)+\ln\sqrt{1+x^2}+c$

(c) $(x+1)\tan^{-1}\left(\frac{1}{x}\right)+c$

(d) $x\tan^{-1}\left(\frac{1}{x}\right)-\ln\left(1+x^2\right)+c$

(e) $\left(\frac{1}{x^2}\right)\tan^{-1}\left(\frac{1}{x}\right)+\ln\sqrt{1+x^2}+c$

11. $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{(1-x^2)^{\frac{5}{2}}}\ dx =$

(a) $\frac{4}{3}$

(b) $\frac{7}{3}$

(c) $\frac{\sqrt{3}}{3}$

(d) $\frac{1}{3}$

(e) $\sqrt{3}$

17. $\int \frac{x^2+x+3}{(x-1)(x^2+2x+2)}\ dx =$

(a) $\ln|x-1| - \tan^{-1}(x+1) + c$

(b) $x + \ln|x-1| + \tan^{-1}(x+1) + c$

(c) $\ln(x-1)^2 - \tan^{-1}(x+1) + c$

(d) $2x + \ln|x-1| - 3\tan^{-1}(x+1) + c$

(e) $\ln|x-1| + 2\tan^{-1}(x+1) + c$

21. $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{\frac{1+\sin x}{1-\sin x}}\ dx =$

(a) $\ln(2+\sqrt{2})$

(b) $\ln(2\sqrt{2}-1)$

(c) $\ln\sqrt{2}$

(d) $\ln(1+2\sqrt{2})$

(e) $\ln(1+\sqrt{2})$

22. The improper integral $\displaystyle\int_{-\infty}^{\infty} e^{-|x|}\ dx$

(a) Converges to 2

(b) Converges to $\dfrac{1}{2}$

(c) Converges to 1

(d) Converges to 0

(e) Diverges

24. The improper integral $\int_0^1 \frac{1}{e^x - 1}\, dx$

(a) Diverges

(b) Converges to $\ln(e-1)$

(c) Converges to $\ln(1-e^{-1})$

(d) Converges to 1

(e) Converges to 0

28. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, $\quad 0 < x < \pi$, we get

$$\int \frac{2\,dx}{\sin x(1+\cos x)} = A\ln\left(\tan\frac{x}{2}\right) + B\tan^2\frac{x}{2} + c \text{ where } A + 2B =$$

(a) 2

(b) 1

(c) 3

(d) 0

(e) 4

(072)

4. $\displaystyle\int \frac{dx}{\sqrt{6x - x^2}} =$

(a) $\sin^{-1} \dfrac{x-3}{3} + c$

(b) $2\sqrt{6x - x^2} + c$

(c) $\ln(6x - x^2) + c$

(d) $\sin^{-1} \dfrac{3-x}{3} + c$

(e) $3\sin^{-1} \dfrac{x-3}{3} + c$

6. $\displaystyle\int_0^{\frac{\pi^2}{4}} \cos\sqrt{x}\, dx =$

(a) 0

(b) $2 - \pi$

(c) $\pi - 2$

(d) $\dfrac{\pi}{2} - 1$

(e) $1 + \dfrac{\pi}{4}$

8. $\int_0^1 \frac{1}{1+e^{-x}}\,dx =$

(a) $\ln(1+e)$

(b) $\ln 2$

(c) e

(d) $\ln\frac{(e+1)}{2}$

(e) $\ln 3$

9. $\int_4^8 (\sqrt{x}+\frac{1}{\sqrt{x}})^2\,dx =$

(a) $32+\ln 2$

(b) $32+\ln 4$

(c) $38+\ln 2$

(d) $38+\ln 4$

(e) $64+\ln 2$

14. $\displaystyle\int \frac{dx}{x^3 - x} =$

(a) $\dfrac{1}{2}\ln\left|x^2 - 1\right| - \ln|x| + c$

(b) $\dfrac{1}{2}\ln\left|\dfrac{x-1}{x+1}\right| - \ln|x| + c$

(c) $\dfrac{1}{2}\ln\left|x^2 - 1\right| + \ln|x| + c$

(d) $\ln\left|x^2 - 1\right| + \ln|x| + c$

(e) $\dfrac{1}{2}\ln\left|\dfrac{x+1}{x-1}\right| + \ln|x| + c$

17. The integral $\int \frac{2}{x + \sqrt[3]{x}}\, dx$ equals

(a) $\ln\left|x + \sqrt[3]{x}\right| + c$

(b) $\ln\left(\frac{2}{3} + x^2\right) + c$

(c) $\ln(2 + e^{-x}) + c$

(d) $2x + 3\ln(x^{2/3} + 1) + c$

(e) $3\ln(x^{2/3} + 1) + c$

18. $\int_0^{\frac{\pi}{2}} \cos^2 x \sin 2x \; dx =$

(a) 0

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $\frac{3}{8}$

(e) -2

19. $\int \frac{1}{x^2(1+x^2)} \, dx =$

(a) $-\frac{1}{x} - \tan^{-1} x + c$

(b) $-\frac{1}{x^2} + \frac{2x}{1+x^2} + c$

(c) $\ln |x| - \tan^{-1} x + c$

(d) $\frac{1}{x} + \ln \left|1 + x^2\right| + c$

(e) $-\frac{1}{x} - \frac{1}{(1+x^2)^2} + c$

22. $\displaystyle\int_0^\infty xe^{-x}dx =$

(a) $\dfrac{1}{e} + 1$

(b) ∞

(c) -1

(d) -2

(e) 1

(071)

1. The value of the integral $\int_0^{\pi/4} \frac{\sin(2x)}{[1+\cos(2x)]^3}\, dx$ is

(a) $\frac{3}{16}$

(b) $\frac{1}{8}$

(c) $\frac{1}{16}$

(d) $\frac{1}{2}$

(e) 1

10. Suppose that $f(1) = 1, f(4) = 7, f'(1) = -1, f'(4) = 3$, and f'' is continuous. Then the value of $\int_1^4 x f''(x)\, dx$ is equal to [Hint: Use integration by parts]

(a) 7

(b) 2

(c) 5

(d) 12

(e) 0

15. The improper integral $\int_0^2 \frac{x^3}{\sqrt{4-x^2}}\,dx$

(a) has the value $\frac{16}{3}$

(b) has the value $\frac{22}{3}$

(c) has the value $\frac{11}{3}$

(d) has the value $\frac{19}{3}$

(e) is divergent

16. The integral $\int \frac{e^{-x}}{e^{-2x} + 3e^{-x} + 2}\, dx$ equals

(a) $\ln\left(\frac{2 + e^{-x}}{1 + e^{-x}}\right) + C$

(b) $\ln\left(\frac{2 + e^{-x}}{1 + e^{x}}\right) + C$

(c) $\ln\left(\frac{2 - e^{-x}}{1 - e^{-x}}\right) + C$

(d) $\ln(2 + e^{-x}) + \ln(1 + e^{-x}) + C$

(e) $\ln(2 - e^{-x}) + \ln(1 - e^{-x}) + C$

17. The value of the integral $\int_1^{16} \frac{1}{\sqrt{x} + \sqrt[4]{x}}\, dx$ is equal to

(a) $2 + 4\ln(1.5)$

(b) $3 - \ln 16$

(c) $2 - 4\ln 3$

(d) $4 + \ln(1.5)$

(e) $\ln(81)$

23. $\displaystyle\int_{1/2}^{3/2} \frac{dx}{5-4x+4x^2}\,dx =$

(a) $\dfrac{\pi}{16}$

(b) $\dfrac{3\pi}{16}$

(c) $\dfrac{3\pi}{4}$

(d) $\dfrac{5\pi}{8}$

(e) $\dfrac{3\pi}{8}$

(063)

1. $\displaystyle\int_0^{\pi/2} \cos^5(x)dx =$

(a) $\dfrac{15}{8}$

(b) 3

(c) $\dfrac{8}{15}$

(d) $-\dfrac{8}{15}$

(e) 0

2. $\displaystyle\int \tan^3(x)\sec^4(x)dx =$

(a) 5

(b) $\dfrac{5}{12}$

(c) 12

(d) $\dfrac{12}{5}$

(e) $-\dfrac{12}{5}$

3. $\displaystyle\int \frac{1}{x^2\sqrt{16-x^2}}\,dx =$

(a) $\dfrac{-\sqrt{16-x^2}}{16x} + C$

(b) $\dfrac{\sqrt{16-x^2}}{16x} + C$

(c) $\dfrac{-\sqrt{16-x^2}}{x} + C$

(d) $\dfrac{-\sqrt{16-x^2}}{16} + C$

(e) $\dfrac{-\sqrt{16-x^2}}{16x^2} + C$

4. Using partial fraction, $\dfrac{1}{x(x^2+1)^2}$ equals

(a) $\dfrac{1}{x} + \dfrac{x}{x^2+1} + \dfrac{x}{(x^2+1)^2}$

(b) $\dfrac{1}{x} - \dfrac{x}{x^2+1} + \dfrac{x}{(x^2+1)^2}$

(c) $\dfrac{-1}{x} - \dfrac{x}{x^2+1} - \dfrac{x}{(x^2+1)^2}$

(d) $\dfrac{1}{x} - \dfrac{x}{x^2+1} - \dfrac{x}{(x^2+1)^2}$

(e) $\dfrac{1}{x} + \dfrac{x}{x^2+1} - \dfrac{x}{(x^2+1)^2}$

8. $\displaystyle\int_1^{\sqrt{2}} x^5\sqrt{x^2-1}\,dx$ equals

(a) $\dfrac{92\pi}{105}$

(b) $\dfrac{105}{92}$

(c) $\dfrac{92}{105}$

(d) $\dfrac{105}{2\pi}$

(e) $\dfrac{105}{92\pi}$

9. $\int_0^1 x^2 e^{-x}\,dx =$

(a) $-5e + 2$

(b) $5e^{-1} + 2$

(c) $-5e^{-1} + 2$

(d) $-5e^{-1} - 2$

(e) $-5e - 2$

10. $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx =$

(a) $\frac{\pi}{4}$

(b) diverges

(c) 0

(d) $\frac{\pi}{2}$

(e) π

(062)

3. The improper integral $\int_0^1 \ln(2x)dx$ is

(a) equal to $+\infty$

(b) convergent and has the value $-1 + \ln 2$

(c) equal to $-\infty$

(d) convergent and has the value $1 + \ln 2$

(e) convergent and has the value $1 - \ln 2$

4. $$\int_0^{\ln\sqrt{3}} \frac{1}{e^x + e^{-x}} dx =$$

(a) $\frac{\pi}{4}$

(b) $\sqrt{3} - 1$

(c) $\frac{\pi}{12}$

(d) $\frac{\pi}{3}$

(e) $e - 1$

6. The indefinite integral $\int_0^{\pi/4} \frac{\sin 3x}{\cos x}\, dx$ is equal to

(a) $1 + \ln\sqrt{2}$

(b) $1 + \ln 2$

(c) $1 + \sqrt{2}$

(d) $1 - \ln 2$

(e) $1 - \ln\sqrt{2}$

8. The integral $\int \frac{dx}{x + x^{3/2}}$ is equal to

(a) $\ln x - 2\ln(\sqrt{x} + 1) + C$

(b) $\ln x - \ln\sqrt{x} + 1 + C$

(c) $\ln x + 2\ln(\sqrt{x} + 1) + C$

(d) $\ln x - 2\ln(\sqrt{x} - 1) + C$

(e) $\ln x + \ln(\sqrt{x} + 1) + C$

9. The value of $\int_0^1 x^3\sqrt{1-x^2}\,dx$ is equal to

(a) $\dfrac{2}{15}$

(b) $\dfrac{2}{3}$

(c) $\dfrac{4}{15}$

(d) $\dfrac{1}{5}$

(e) $\dfrac{8}{15}$

10. The integral $\displaystyle\int \frac{dx}{1-\sin x}$ is equal to

(a) $\cot x + \csc x + C$

(b) $\tan x - \sec x + C$

(c) $\tan x + \csc x + C$

(d) $\tan x + \sec x + C$

(e) $\sec x - \tan x + C$

13. The improper integral $\int_e^\infty \frac{dx}{x(\ln x)^2}$

(a) diverges

(b) converges to 0

(c) converges to $\frac{1}{e}$

(d) converges to e

(e) converges to 1

1. The integral $\int_1^e 9x^2 \ln x dx$ is equal to

(a) $2e^3 + 3$
(b) $2e^3 - 2$
(c) $3e^3 - 1$
(d) $2e^3 + 1$
(e) $3e^3 + 1$

2. The integral $\int \sin 2\theta \cos \theta d\theta$ is equal to

(a) $-\frac{2}{3} \sin^3 \theta + C$
(b) $-\frac{1}{3} \cos 3\theta + C$
(c) $\frac{1}{3} \sin 3\theta + C$
(d) $-\frac{2}{3} \cos^3 \theta + C$
(e) $\frac{1}{4} \sin^2 2\theta + C$

3. The integral $\int\limits_0^1 \frac{dx}{(x^2+1)^{\frac{3}{2}}}$ is equal to

(a) $\frac{\sqrt{2}}{2} - 1$
(b) $\frac{\sqrt{2}}{2} + 1$
(c) $\frac{\sqrt{2}}{2}$
(d) $\sqrt{2}$
(e) $\sqrt{2} - 1$

4. The integral $\int\limits_0^1 \frac{x^2-5x+7}{x^2-5x+6}dx$ is equal to

(a) $1-\ln\frac{4}{3}$
(b) $2+\ln\frac{3}{4}$
(c) $1-\ln\frac{2}{3}$
(d) $1+\ln\frac{4}{3}$
(e) $1+\ln\frac{2}{3}$

5. The integral $\int\limits_{-\infty}^{\infty} x^2e^{-x^3}dx$

(a) converges to $\frac{1}{3}$.
(b) converges to $\frac{1}{6}$.
(c) diverges.
(d) converges to 0.
(e) converges to $\frac{2}{3}$.

12. The integral $\int\limits_0^{\frac{\pi^2}{9}} \sin\sqrt{x}dx$ is equal to

(a) $2\sin\frac{\pi^2}{9}-\frac{2\pi^2}{9}\cos\frac{\pi^2}{9}$
(b) $\frac{\pi\sqrt{3}}{2}-1$
(c) $1-\cos\frac{\pi^2}{9}$
(d) $\frac{\sqrt{3}}{2}-\frac{\pi}{6}$
(e) $\sqrt{3}-\frac{\pi}{3}$

Answer Key :

Question	Answer
2(092)	A
3(092)	A
4(092)	A
11(092)	A
25(092)	A
26(092)	A
1(091)	A
2(091)	A
6(091)	A
8(091)	A
12(091)	A
18(091)	A
22(091)	A
25(091)	A
2(083)	E
9(083)	B
11(083)	B
14(083)	E

27(083)	C
28(083)	B
2(082)	A
5(082)	A
7(082)	A
8(082)	A
10(082)	A
26(082)	A
2(081)	B
3(081)	C
6(081)	C
7(081)	B
8(081)	B
10(081)	C
13(081)	B
16(081)	D
18(081)	A
5(073)	A
9(073)	A
11(073)	A
17(073)	A
21(073)	A
22(073)	A
24(073)	A
28(073)	A
4(072)	A

6(072)	C
8(072)	D
9(072)	A
14(072)	A
17(072)	E
18(072)	C
19(072)	A
22(072)	E
1(071)	A
10(071)	A
15(071)	A
16(071)	A
17(071)	A
23(071)	A
1(063)	--
2(063)	--
3(063)	--
4(063)	--
8(063)	--
9(063)	--
10(063)	--
3(062)	B
4(062)	C
6(062)	E
8(062)	A
9(062)	A

10(062)	D
13(062)	E
1(061)	D
2(061)	D
3(061)	C
4(061)	D
5(061)	C
12(061)	E

Old Exam 8.1 :

(092)

5. The length of the curve $x = \frac{2}{3} y^{3/2}$ from $y = 0$ to $y = 3$ is

(a) $\frac{14}{3}$

(b) $\frac{11}{3}$

(c) $\frac{17}{3}$

(d) 5

(e) 4

(091)

26. The length of the curve

$$y = 10 + 2x^{3/2}\,, \qquad 0 \leq x \leq 1$$

is equal to

(a) $\frac{2}{27}(10\sqrt{10} - 1)$

(b) $\frac{1}{27}(\sqrt{10} - 1)$

(c) $\frac{2}{9}$

(d) $\frac{2}{9}(10\sqrt{10} - 3)$

(e) $\frac{5}{27}(\sqrt{10} - 10)$

17. The length of the curve

$$y = \int_0^x \sqrt{9\sin^2 t - 1}\, dt, \quad 0 \le x \le \frac{\pi}{2}$$

is equal to

(a) $\frac{1}{3}$

(b) $\sqrt{3}$

(c) 3

(d) $\frac{3}{2}$

(e) 9

(082)

27. The arc length of the curve $y = x^2 - \frac{1}{8}\ln x, \quad 1 \le x \le 3$ is equal to

(a) $8 + \frac{1}{8}\ln 3$

(b) $3 + \frac{1}{3}\ln 8$

(c) $8 + \ln 3$

(d) $3 + \ln 8$

(e) $-8 + \frac{1}{8}\ln 3$

(081)

17. The length of the curve $y = \ln(\cos x)$, $\quad 0 \leq x \leq \dfrac{\pi}{4}$, is

(a) $2 + \sqrt{2}$

(b) $\ln(1 + \sqrt{2})$

(c) $1 + \sqrt{2}$

(d) $\ln(\sqrt{2})$

(e) $\ln(\sqrt{2} + \sqrt{3})$

(073)

27. An electric cable is hung between two towers that are 200 feet apart. If the cable takes the shape of a curve whose equation is

$$y = 50\cosh(x/50), \quad -100 \le x \le 100,$$

then the length of the cable between the two towers is equal to

(a) $50\left(e^2 - e^{-2}\right)$

(b) $100\left(e^2 + e^{-2}\right)$

(c) $50\left(e - e^{-1}\right)$

(d) $100\left(e + e^{-1}\right)$

(e) $50\left(e^2 - e^{-2}\right)^2$

(072)

12. The length of the curve $y = \ln(\cos x)$, $0 \le x \le \frac{\pi}{3}$ is

(a) $\ln(2 + \sqrt{3})$

(b) $\ln 3$

(c) $\csc\left(\frac{\pi}{2}\right) - \csc\left(\frac{\pi}{3}\right)$

(d) $\ln\left(\frac{3}{2}\right)$

(e) $-\frac{1}{2}$

(071)

14. The length of the curve $y = \ln(\sec x)$, $\quad 0 \le x \le \frac{\pi}{4}$, is

(a) $\ln(1 + \sqrt{2})$

(b) $\ln(\sqrt{2})$

(c) $1 + \sqrt{2}$

(d) $\ln(\sqrt{2} + \sqrt{3})$

(e) $2 + \sqrt{2}$

(063)

6. The length L of the graph of $f(x) = \ln x - \frac{1}{8}x^2$ for $1 \leq x \leq e$ is equal to

(a) $1 + \frac{e^2 - 1}{8}$

(b) $\frac{e^2 - 1}{8}$

(c) $\pi + \frac{e^2 - 1}{8}$

(d) $1 - \frac{e^2 + 1}{8}$

(e) $\pi + \frac{e^2 + 1}{8}$

(062)

2. The length of the curve $y = \frac{2}{3}(x^2 - 1)^{3/2}, \quad 1 \leq x \leq 3$ is equal to

(a) $\frac{15}{4}$

(b) $\frac{46}{3}$

(c) 4

(d) 15

(e) $\frac{22}{3}$

(061)

20. The length of the curve $y = \frac{x^2}{2} - \frac{\ln x}{4}$, $2 \leq x \leq 4$ is equal to

(a) $6 + \frac{\ln 2}{4}$
(b) $6 - \ln 2$
(c) $6 - \frac{\ln 2}{4}$
(d) $6 + \ln 2$
(e) $6 - \ln 3$

Answer Key :

Question	Answer
5 (092)	A
26(091)	A

17 (083)	C
17 (081)	B
27 (073)	A
12 (072)	A
14(071)	A
6(063)	--
2(062)	B
20(061)	A

Old Exam 8.2 :

(092)

12. The area of the surface obtained by rotating the curve $y = \sqrt{x}$, $2 \leq x \leq 6$, about the x-axis, is equal to

(a) $\frac{49\pi}{3}$

(b) $\frac{79\pi}{3}$

(c) 49π

(d) 79π

(e) $\frac{101\pi}{6}$

(091)

28. The area of the surface obtained by rotating the curve

$$y = x^5, \quad 1 \le x \le 32$$

about the x-axis is given by

(a) $\int_1^{32} 2\pi x^5 \sqrt{1 + 25x^8}\, dx$

(b) $\int_1^{32} 2\pi x^5 \sqrt{1 + 5x^4}\, dx$

(c) $\int_1^{2} 2\pi y \sqrt{1 + 25x^8}\, dy$

(d) $\int_1^{2} 2\pi \sqrt[5]{y} \cdot \sqrt{1 + \frac{1}{25} y^{-8/5}}\, dy$

(e) $\int_1^{32} 2\pi x \sqrt{1 + 25x^8}\, dx$

(083)

16. The area of the surface generated by revolving the curve

$$y = \sqrt{x}\,, \quad 0 \le x \le 2$$

about the x-axis is equal to

(a) $\frac{2\pi}{3}(3\sqrt{3} - 2)$

(b) $\frac{2\pi}{3}(3\sqrt{3} - 16)$

(c) 13π

(d) $\frac{19\pi}{3}$

(e) $\frac{13\pi}{3}$

(082)

11. The area of the surface obtained by rotating the curve $y = x^3,\ \ 0 \le x \le 1$ about the x-axis is

(a) $\dfrac{\pi}{27}(10\sqrt{10} - 1)$

(b) $\dfrac{\pi}{27}(145\sqrt{145} - 1)$

(c) $\dfrac{\pi}{18}(10\sqrt{10} - 1)$

(d) $\dfrac{\pi}{18}(145\sqrt{145} - 1)$

(e) $\dfrac{\pi}{27}$

(081)

25. If the curve $x = \frac{1}{3}\sqrt{4-9y^2}$, $0 \le y \le \frac{1}{3}$, is rotated about the y-axis, then the area of the resulting surface is equal to

(a) $\frac{4\pi}{9} \sin^{-1}\left(\frac{2}{3}\right)$

(b) $\frac{2\pi}{3} \sin^{-1}\left(\frac{1}{3}\right)$

(c) $\frac{8\pi}{3}$

(d) $\frac{\pi}{9}$

(e) $\frac{4\pi}{9}$

(073)

8. The area of the surface obtained by rotating the curve $y = \sqrt{3-x^2}$, $0 \le x \le 1$, about the x-axis is equal to

(a) $2\pi\sqrt{3}$

(b) $\frac{4\pi\sqrt{3}}{3}$

(c) $\frac{4\sqrt{3}}{2}$

(d) $6\pi\sqrt{3}$

(e) $\frac{\pi\sqrt{3}}{6}$

(072)

11. The area of the surface of the solid obtained by rotating the curve $y = \sqrt{1+e^x}$, $0 \le x \le 1$ about the x-axis is equal to

(a) $\pi(e+1)$

(b) $\pi(2e+1)$

(c) $2\pi(e-1)$

(d) πe

(e) $\pi(e-1)$

(071)

21. The integral for the area of the surface obtained by rotating the curve $y = \tan x$ from $(0,0)$ to $\left(\frac{\pi}{4}, 1\right)$ about the y-axis is

(a) $2\pi \int_0^{\pi/4} x\sqrt{1+\sec^4 x}\, dx$

(b) $2\pi \int_0^{\pi/4} x\sqrt{1+\tan^4 x}\, dx$

(c) $2\pi \int_0^{\pi/4} \tan x\sqrt{1+\sec^4 x}\, dx$

(d) $2\pi \int_0^{1} y\sqrt{1+\frac{1}{1+y^2}} dy$

(e) $2\pi \int_0^{\pi/4} \tan x\sqrt{1-\tan^2 x}\, dx$

(063)

7. If the curve of $f(x) = \sqrt{1-x^2}$ for $0 \le x \le \frac{1}{2}$ revolves about the x-axis then the area of the surface of revolution is equal to

(a) 2π

(b) $\frac{\pi}{2}$

(c) π

(d) $\frac{\pi}{4}$

(e) $\frac{\pi}{8}$

(062)

24. An integral for the area of the surface obtained by rotating the curve $y = \sec x$,
$0 \leq x \leq \dfrac{\pi}{4}$ about the y-axis is

(a) $\displaystyle\int_0^{\pi/4} 2\pi y\sqrt{1 + (\sec^{-1} x \tan^{-1} x)^2}\, dx$

(b) $\displaystyle\int_0^{\pi/4} 2\pi \sec^{-1} y\sqrt{1 + \frac{1}{y^2(y^2+1)}}\, dy$

(c) $\displaystyle\int_1^{\sqrt{2}} 2\pi y\sqrt{1 + \frac{1}{y^2(y^2-1)}}\, dy$

(d) $\displaystyle\int_1^{\sqrt{2}} 2\pi x\sqrt{1 + (\sec x \tan x)^2}\, dx$

(e) $\displaystyle\int_0^{\pi/4} 2\pi x\sqrt{1 + (\sec x \tan x)^2}\, dx$

(061)

21. The area of the surface obtained by rotating the curve $x = \frac{1}{3}(y^2+2)^{\frac{3}{2}}$, $1 \leq y \leq 2$, about the $x-$axis is equal to

(a) 21π
(b) $21\pi/2$
(c) $21\pi/4$
(d) $20\pi/3$
(e) 20π

Answer Key :

Question	Answer
12 (092)	A
28(091)	A
16(083)	E
11 (082)	A
25 (081)	E
8 (073)	A
11 (072)	A
21(071)	A

7(063)	--
24(062)	E
21(061)	B

Old Exam 11.1 :

(092)

Q.3. $(6-\ points)$. Determine whether the sequence $\left\{\dfrac{e^n+e^{-n}}{e^{2n}-1}\right\}$ converges or diverges.

If it converges, find the limit.

Q.9. $(6-\ points)$. Determine whether the sequence

$$\left(\frac{2}{4}-\frac{1}{3}\right),\left(\frac{4}{5}-\frac{1}{5}\right),\left(\frac{6}{6}-\frac{1}{7}\right),\left(\frac{8}{7}-\frac{1}{9}\right),\cdots$$

converges or diverges. If it converges, find the limit.

(091)

4. [**6 points**] Determine whether the sequence $\left\{\dfrac{(-1)^n\sqrt{n}}{n+7}\right\}_{n=1}^{+\infty}$ is convergent or divergent.

If it is convergent, find its limit.

(083)

4. [**6 points**] Find a formula for the general term a_n of the sequence and determine whether the sequence converges or diverges:

$$\left\{\frac{7}{2\cdot 3}, \frac{7\cdot 2^2}{3\cdot 4}, \frac{7\cdot 3^2}{4\cdot 5}, \frac{7\cdot 4^2}{5\cdot 6} \cdots\right\}$$

(082)

Q.6. Determine whether the sequence converges or diverges. If it converges, find its limit.

(a) . (4 points) . $a_n = \dfrac{1+\sin 2n}{1+\sqrt{n}}$.

Q.6. (b) . (4 points) . $a_n = \ln\sqrt{n+1} - \dfrac{1}{2}\ln n$.

(081)

5. [**6 points**] Determine whether the sequence $\left\{\dfrac{\sin(3n)}{3n+1}\right\}_{n=1}^{+\infty}$ converges or diverges. If it converges, find its limit.

6. Consider the series $\displaystyle\sum_{n=1}^{+\infty} \frac{\ln(n+2)-\ln(n+1)}{\ln(n+1)\cdot\ln(n+2)}$.

   (a) [**5 points**] Find a formula for S_n, the sequence of partial sums.

(072)

9. (6 points) Given the following sequence:

$$\left(1-\frac{1}{2}\right), \left(\frac{1}{2}-\frac{1}{3}\right), \left(\frac{1}{3}-\frac{1}{4}\right), \left(\frac{1}{4}-\frac{1}{5}\right), \ldots$$

(a) Find the general term of the sequence.

(b) Show that the sequence is convergent.

<u>Final Exam 11.1 :</u>

(092)

27. The limit of the sequence defined by $s_n = \frac{1}{n}\sin\left(\frac{n\pi}{4}\right) + \frac{(2n+6)}{(n+1)}$

(a) is equal to 2

(b) is equal to 0

(c) is equal to 1

(d) oscillates between -1 and 1

(e) is ∞

(091)

15. The sequence $\left\{\frac{(2n-1)!\cdot(5n^2+2n)}{(2n+1)!}\right\}_{n=1}^{+\infty}$

(a) converges and its limit is $\frac{5}{4}$

(b) converges and its limit is 5

(c) converges and its limit is 1

(d) converges and its limit is $\frac{5}{2}$

(e) is divergent

(083)

4. The sequence $\left\{2-\frac{\cos n}{2^n}\right\}_{n=1}^{+\infty}$

(a) converges to 3

(b) converges to 1

(c) diverges

(d) converges to -1

(e) converges to 2

(082)

22. Let us consider the sequence $\{\tan^{-1}(-3n)\}$. Then the sequence

(a) converges and its limit is $-\frac{\pi}{2}$

(b) converges and its limit is 0

(c) converges and its limit is $\frac{\pi}{2}$

(d) diverges

(e) converges and its limit is -1

(081)

15. The sequence $\left\{ \dfrac{(-1)^n n^2}{n^2+n+1} \right\}_{n=1}^{+\infty}$

(a) converges to 1

(b) diverges

(c) converges to -1

(d) converges to -2

(e) converges to 0

(073)

15. The sequence $\left\{(2n+1)\sin\frac{7}{n}\right\}$

(a) Converges to 14

(b) Converges to $\frac{7}{2}$

(c) Converges to 0

(d) Converges to $\frac{2}{7}$

(e) Diverges

(072)

24. The sequence $\left\{\left(1+\frac{2}{n}\right)^n\right\}_{n=1}^{\infty}$

(a) converges to $\sqrt{e}$

(b) converges to e^2

(c) converges to e

(d) diverges

(e) converges to 2

(071)

6. The sequence $\{(2-e)^n\}_{n=1}^{+\infty}$

(a) converges to 0

(b) converges to $-e$

(c) converges to $\dfrac{2}{e}$

(d) converges to 2

(e) diverges

(063)

14. The sequence $\left\{(-1)^n \dfrac{3n^2+5}{n^3-n^2+1}\right\}_{n=1}^{\infty}$:

(a) converges to 0
(b) converges to -1
(c) converges to 1
(d) converges to 3
(e) diverges

18. The limit of the sequence $\{n\sqrt[n]{e} - n\}_{n=1}^{+\infty}$

(a) is equal to 1

(b) is equal to 0

(c) is equal to e

(d) does not exist

(e) is equal to -2

22. The sequence $a_n = \frac{\sqrt{n}}{1+\sqrt{n}}$

(a) converges and the limit is 1
(b) diverges
(c) converges and the limit is $\sqrt{2}$
(d) converges and the limit is $\frac{\sqrt{2}}{1+\sqrt{2}}$
(e) converges and the limit is 0

Answer Key :

Question	Answer
27 (092)	A
15(091)	A
4 (083)	E
22 (082)	A
15 (081)	B
15 (073)	A
24 (072)	B
6(071)	A
14(063)	--

18(062)	A
22(061)	A

Old Exam 11.2 :

(092)

1. $$\sum_{n=1}^{\infty} \frac{1+(-2)^n}{3^n} =$$

(a) 0.1

(b) 0.01

(c) 0.001

(d) 1.1

(e) 1.01

15. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)} =$

(a) $\frac{11}{6}$

(b) $\frac{3}{2}$

(c) $\frac{4}{3}$

(d) $\frac{5}{3}$

(e) 1

(091)

16. The series $\sum_{n=1}^{+\infty} (-1)^n \frac{n+27}{n+28}$ is

(a) divergent

(b) absolutely convergent

(c) conditionally convergent

(d) divergent by the integral test

(e) convergent by the ratio test

23. The series $\sum_{n=1}^{+\infty} \frac{2^n + (-4)^n}{8^n}$

(a) converges and its sum is 0

(b) converges and its sum is $\frac{2}{3}$

(c) converges and its sum is $\frac{3}{8}$

(d) converges and its sum is $\frac{3}{4}$

(e) diverges

(083)

5. The series $\sum_{n=1}^{+\infty} \frac{n^2-1}{n^2+1}$ is

(a) divergent by the ratio test

(b) convergent by the integral test

(c) divergent

(d) convergent

(e) convergent by comparing it with a suitable p-series

(082)

12. The sum of the series $\sum_{n=1}^{\infty} [\frac{3}{n(n+1)} + \frac{1}{2^n}]$ is equal to

(a) 4

(b) 3

(c) 2

(d) 1

(e) 5

(081)

11. The series $\sum_{n=1}^{+\infty} \frac{2^n + (-1)^{n-1}}{3^n}$

(a) diverges

(b) converges and its sum is $\frac{3}{4}$

(c) converges and its sum is 2

(d) converges and its sum is $\frac{2}{3}$

(e) converges and its sum is $\frac{9}{4}$

(073)

4. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n + 2^{n+1}}{3^n} =$

(a) $\dfrac{27}{4}$

(b) $\dfrac{9}{4}$

(c) $\dfrac{11}{4}$

(d) $\dfrac{31}{4}$

(e) $\dfrac{23}{4}$

(072)

20. The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

(a) is a convergent geometric series

(b) is a convergent p-series

(c) converges to $\ln 2$

(d) is divergent

(e) converges to 1

(071)

7. If the n-th partial sum of a series $\sum_{n=1}^{+\infty} a_n$ is $s_n = 2 - \frac{(-1)^n}{n^2}$, then the series $\sum_{n=1}^{+\infty} a_n$

(a) converges and its sum is 2

(b) converges and its sum is 1

(c) diverges

(d) converges and its sum is $\frac{3}{2}$

(e) converges and its sum is $\frac{1}{2}$

8. The series $\sum_{n=1}^{+\infty} \frac{(-3)^{n+1}}{2^{3n}}$

(a) converges and its sum is $\frac{9}{11}$

(b) converges and its sum is $\frac{9}{5}$

(c) converges and its sum is $\frac{-24}{11}$

(d) converges and its sum is $\frac{-3}{11}$

(e) diverges

(063)

15. The series $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$:

(a) converges to 1

(b) converges to 2

(c) converges to e

(d) converges to $\frac{1}{e}$

(e) diverges

16. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2+n} =$

(a) 0

(b) 1

(c) 2

(d) 3

(e) ∞

17. $\displaystyle\sum_{n=1}^{\infty} \left(\ln(n) - \frac{1}{n}\right) =$

(a) $-\infty$

(b) 0

(c) 1

(d) 2

(e) ∞

18. $\displaystyle\sum_{n=4}^{\infty}\left(\frac{-1}{e}\right)^{n-1} =$

(a) $\dfrac{1}{e}$

(b) $\dfrac{e}{1+e}$

(c) $\dfrac{1}{e(e-1)}$

(d) $\dfrac{-1}{e^2(e+1)}$

(e) The series diverges

(062)

20. The series $\displaystyle\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^{-k}$

(a) converges to e

(b) diverges

(c) converges to $\dfrac{1}{e}$

(d) converges to 0

(e) converges to 1

(061)

23. The series $\sum_{n=1}^{\infty} \frac{3^n+2^n}{6^n}$ is

(a) convergent and the sum is 7
(b) convergent and the sum is 3
(c) divergent
(d) convergent and the sum is $5/2$
(e) convergent and the sum is $3/2$

Answer Key :

Question	Answer
1 (092)	A
15 (092)	A
16(091)	A
23 (091)	A
5(083)	C
12 (082)	A

11 (081)	E
4 (073)	A
20 (072)	E
7(071)	A
8 (071)	A
15 (063)	--
16(063)	--
17(063)	--
18(063)	--
20(062)	B
23(061)	E

Old Exam 11.3 :

(092)

6. The set of all values of P, in interval notation, for which the series $\sum_{n=1}^{\infty} n(1+n^2)^P$ is convergent, is

(a) $(-\infty, -1)$

(b) $(0, \infty)$

(c) $(-\infty, 0)$

(d) $(1, \infty)$

(e) $(-\infty, 1)$

16. The error of using the sum of the first 10 terms to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4+1}}$ can be estimated as a number in the interval

(a) $(0, 0.1)$

(b) $(0.1, 0.2)$

(c) $(0.2, 0.3)$

(d) $(0.3, 0.6)$

(e) $(0.4, 0.5)$

(082)

21. Let us consider the series $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}$. Then the integral test

(a) implies that the series converges

(b) is not applicable because of the continuity condition

(c) implies that the series diverges

(d) is not applicable because of the decreasing condition

(e) is not applicable because of the positivity condition

(081)

1. Which one of the following is **TRUE**?

(a) $\sum_{n=1}^{+\infty} \frac{1}{n^{e-2}}$ is convergent

(b) $\sum_{n=1}^{+\infty} \frac{1}{n^{0.999}}$ is convergent

(c) $\sum_{n=1}^{+\infty} \frac{1}{n^{\pi/4}}$ is divergent

(d) $\sum_{n=1}^{+\infty} \frac{1}{n^{\sqrt{2}}}$ is divergent

(e) $\sum_{n=1}^{+\infty} \frac{1}{n^{\pi-2}}$ is divergent

9. The series $1 + \dfrac{1}{2^2\sqrt{2}} + \dfrac{1}{3^2\sqrt{3}} + \dfrac{1}{4^2\sqrt{4}} + \cdots$ is

(a) a convergent p-series with $p = \dfrac{5}{2}$

(b) a divergent series

(c) a convergent p-series with $p = 2$

(d) a divergent series by the integral test

(e) a divergent p-series with $p = \dfrac{1}{2}$

12. The series $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{n \ln n}$

(a) diverges by the integral test

(b) converges by the integral test

(c) converges by the comparison test with $b_n = \dfrac{1}{n}$

(d) diverges by the comparison test with $b_n = \dfrac{1}{n^2}$

(e) diverges by the ratio test

(062)

22. If you want to use the integral test to test the series $\sum_{n=1}^{\infty} n\,e^{-n^2}$ for convergence, then your conclusion is

(a) the integral test is not applicable in this case

(b) the integral converges to $\frac{1}{2e}$

(c) the integral converges to $3e$

(d) the integral diverges

(e) the integral converges to $\frac{1}{e^2}$

Answer Key :

Question	Answer
6 (092)	A
16 (092)	A
21 (082)	A
1 (081)	C
9(071)	A
12 (071)	A
22(062)	B

Old Exam 11.4 :

(092)

14. The series $\sum_{n=1}^{\infty} \dfrac{1+\sin^2 n}{n+n\sqrt{n}}$

(a) converges by the comparison test

(b) diverges by the integral test

(c) is a convergent geometric series

(d) is a convergent p-series

(e) diverges by the root test

(082)

3. The series $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$ is

(a) divergent

(b) convergent to 0.1

(c) convergent to 100

(d) convergent to 0.01

(e) convergent to 0.001

(073)

10. The series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{\sqrt[3]{n^4+5}}$

(a) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$

(b) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

(c) Converges by the integral test

(d) Diverges by the test for divergence

(e) Diverges by the comparison test with $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$

(071)

18. The series $\sum_{n=1}^{+\infty} n \sin\left(\frac{1}{n}\right)$

(a) diverges

(b) converges and its sum is 1

(c) converges and its sum is 0

(d) converges

(e) converges and its sum is $\frac{1}{3}$

19. The series $\sum_{n=1}^{+\infty} \frac{n^2+1}{n^5+n^4+1}$ is

(a) convergent

(b) divergent

(c) convergent and its sum is 1

(d) divergent by the test of divergence

(e) convergent by the ratio test

(061)

6. The series $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$

(a) diverges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$
(b) converges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$
(c) converges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}}$
(d) converges by comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}}$
(e) diverges because $\lim_{n\to\infty} \frac{1}{n\sqrt[n]{n}} = \frac{1}{e}$.

Answer Key :

Question	Answer
14 (092)	A
3 (082)	A
10 (073)	A
18(071)	A
19 (071)	A
6(061)	A

Old Exam 11.5 :

(092)

17. The smallest number of terms, needed in order to find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$ with |error| less than 0.001, is

(a) 100

(b) 50

(c) 10

(d) 150

(e) 200

(091)

17. The smallest number of terms of the series $\sum_{n=1}^{+\infty} \frac{(-1)^n}{(2n+1)^4}$ that we need to add so that $|\text{error}| < 0.0001$ is

(a) 4

(b) 40

(c) 400

(d) 10

(e) 22

(083)

22. The error in approximating the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n+2}}$ by the sum of the first 29 terms is less than or equal to

(a) $\frac{2}{5}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{5}$

(e) $\frac{1}{\sqrt[5]{33}}$

(082)

4. For what values of p, is the series $\sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n^{p-4}}$ convergent?

(a) $p > 4$

(b) $p > 1$

(c) $p \geq 4$

(d) $p < 4$

(e) $p \leq 4$

14. Which one of the following statements is TRUE about the alternating series $\sum_{n=1}^{+\infty} (-1)^{n-1} a_n$, where $a_n = \dfrac{2n}{3n+1}$?

(a) The series is divergent

(b) The series is absolutely convergent

(c) The series is conditionally convergent

(d) $a_{n+1} \leq a_n$ for all n

(e) $\sum_{n=1}^{+\infty} (-1)^{n-1} a_n = \dfrac{2}{3}$

15. How many terms of the series $\sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n^3}$ do we need to add in order to ensure that the sum is accurate to within 0.001? (minimum number of terms)

(a) 9

(b) 10

(c) 100

(d) 99

(e) 1000

(081)

20. The smallest number of terms of the series $\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ that we need to add so that $|\text{error}| < 0.1$ is

(a) 80

(b) 90

(c) 60

(d) 70

(e) 100

(073)

7. The set of all values of p for which the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n-1)^{5p-1}}$ converges, is given by the interval

(a) $\left(\frac{1}{5}, \infty\right)$

(b) $(1, \infty)$

(c) $[1, \infty)$

(d) $\left[\frac{1}{5}, \infty\right)$

(e) $(0, \infty)$

26. For the convergent alternating series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^3}$, what is the smallest number of terms needed to guarantee that S_n approximates S within $\frac{1}{125} \times 10^{-6}$?

(a) 499

(b) 599

(c) 488

(d) 198

(e) 408

(072)

16. For the convergent alternating series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}$, what is the smallest number of terms needed to guarantee that S_n is within 1×10^{-8} of the actual sum S?

(a) 100

(b) 99

(c) 1000

(d) 10

(e) 80

(071)

13. The error in approximating the sum of the series $\sum_{n=1}^{+\infty} \frac{(-1)^n n}{5^n}$ by the sum of the first four terms is less than or equal to

(a) $\frac{1}{5^4}$

(b) $\frac{1}{4 \cdot 5^4}$

(c) $\frac{6}{5^6}$

(d) $\frac{1}{5^5}$

(e) $\frac{4}{5^5}$

20. The series $\sum_{n=1}^{+\infty} \frac{(-1)^n 3n}{4n-1}$ is

(a) divergent

(b) convergent

(c) absolutely convergent

(d) conditionally convergent

(e) neither convergent nor divergent

(062)

17. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$

(a) converges for $p \leq 0$

(b) converges for all real numbers p

(c) diverges for all p

(d) converges only for $p = 0$

(e) converges for $p > 0$

Answer Key :

Question	Answer
17 (092)	A
17(091)	A
22 (083)	C
4(082)	A
14 (082)	A
15 (082)	A
20 (081)	E
7 (073)	A
26(073)	A
16 (072)	B
20(071)	A
17(062)	E

Old Exam 11.6 :

(092)

18. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(a) converges conditionally

(b) converges absolutely

(c) diverges

(d) is a convergent geometric series

(e) is a divergent geometric series

28. The series $\sum_{n=1}^{\infty}(-1)^n \frac{n!+n}{(n+1)!}$

(a) converges conditionally

(b) converges absolutely

(c) diverges by alternating series test

(d) diverges because $\sum_{n=1}^{\infty} \frac{n!+n}{(n+1)!}$ diverges

(e) converges because $\sum_{n=1}^{\infty} \frac{n!+n}{(n+1)!}$ converges

(091)

13. The series $\sum_{n=1}^{+\infty} e^{-n} \cdot n!$

(a) diverges by the ratio test

(b) converges by the ratio test

(c) diverges by the integral test

(d) converges by the comparison test

(e) converges by the test for divergence

19. Which one of the following statements is **TRUE**:

(a) If $a_n > 0$ for all n and $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{3}$, then $\sum\limits_{n=1}^{+\infty} a_n$ is convergent

(b) The series $\sum\limits_{n=1}^{+\infty} n^{-\pi}$ is divergent

(c) If $0 < a_n \leq b_n$ for all n and $\sum\limits_{n=1}^{+\infty} b_n$ diverges, then $\sum\limits_{n=1}^{+\infty} a_n$ diverges

(d) If $\lim\limits_{n\to+\infty} a_n = 0$, then $\sum\limits_{n=1}^{+\infty} a_n$ is convergent

(e) If $\lim\limits_{n\to+\infty} \sqrt[n]{|a_n|} = 1$, then $\sum\limits_{n=1}^{+\infty} a_n$ is divergent

21. The series $\sum\limits_{n=1}^{+\infty} \frac{n^{2n}}{(1+2n^2)^n}$

(a) converges by the root test

(b) diverges by the root test

(c) is a series with which the root test is inconclusive

(d) diverges by the test of divergence

(e) diverges by the comparison test

(083)

21. The series $\sum_{n=1}^{+\infty} \frac{(n+1)!}{3^{n-1} \cdot [5 \cdot 7 \cdot 9 \cdots (2n+3)]}$ is

(a) divergent by the test of divergence

(b) convergent by the ratio test

(c) convergent by the test of divergence

(d) divergent by the ratio test

(e) a series with which the ratio test is inconclusive

25. The series $\sum_{n=1}^{+\infty} \frac{(-1)^n}{n+28}$ is

(a) divergent

(b) absolutely convergent

(c) conditionally convergent

(d) convergent by the ratio test

(e) neither convergent nor divergent

(082)

16. By applying the Ratio Test to the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}$, we conclude that the

(a) test fails

(b) series is convergent

(c) series is absolutely convergent

(d) series is conditionally convergent

(e) series is divergent

18. By applying the Root Test to the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{3^{1+3n}}$, we conclude that the

(a) series is divergent

(b) test fails

(c) series is convergent

(d) series is absolutely convergent

(e) series is conditionally convergent

(081)

19. The series $\sum_{n=1}^{+\infty} (-1)^n \frac{2^{2n} \cdot (n+1)^2}{n!}$ is

(a) convergent by the integral test

(b) a series with which the ratio test is inconclusive

(c) divergent by the ratio test

(d) convergent by the ratio test

(e) divergent by the test for divergence

(073)

3. The series $\sum_{n=1}^{\infty}\left(\frac{n^3+1}{2n^3+n}\right)^n$

(a) Converges by the Root test

(b) Diverges by the Root test

(c) Diverges by the Ratio test

(d) is a convergent geometric series

(e) Diverges by the limit comparison test

18. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, n^5\, 6^n}{(n+1)!}$

(a) is absolutely convergent

(b) is conditionally convergent

(c) is divergent by the ratio test

(d) is divergent by the test for divergence

(e) is convergent by the integral test

20. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(a) Converges conditionally

(b) Converges absolutely

(c) Diverges

(d) Converges by the integral test

(e) Converges by the root test

(072)

25. The series $\sum_{k=0}^{\infty} \frac{(-1)^k k!}{e^k}$ is

(a) convergent by the root test

(b) convergent to e^{10}

(c) convergent by the ratio test

(d) divergent

(e) convergent to e^3

(e) is absolutely divergent

(071)

24. The series $\sum_{n=1}^{+\infty}\left(\frac{1+\ln n}{n^2+3}\right)^n$ is

(a) convergent by the root test

(b) divergent by the root test

(c) a convergent geometric series

(d) a series with which the root test is inconclusive

(e) divergent by the test of divergence

(063)

19. The series $\sum_{n=1}^{\infty}\frac{-\ln(n)}{n^2+3}$:

(a) converges absolutely
(b) converges conditionally
(c) converges to 3
(d) converges to e
(e) diverges

20. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$:

(a) converges absolutely

(b) converges to -1

(c) converges to 3

(d) converges to 2

(e) converges conditionally

25. The series $\displaystyle\sum_{n=3}^{\infty}(-1)^n \left(\frac{n^3+2n-1}{3n^3+3n^2-6}\right)^n$:

(a) converges absolutely

(b) converges conditionally

(c) diverges

(d) $=\infty$

(e) $=-\infty$

(062)

15. The series $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n!}$

(a) converges conditionally

(b) converges absolutely

(c) is convergent to 0

(d) is convergent to $\frac{1}{e}$

(e) is divergent

19. The series $\sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$

(a) diverges by the test for divergence

(b) diverges by comparison test

(c) converges

(d) $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$

(e) $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| > 1$

(061)

8. The series $\sum_{n=1}^{\infty} (-1)^n \, \frac{n^2\, 2^n}{n\,!}$

(a) converges conditionally.
(b) diverges by ratio test.
(c) diverges by integral test.
(d) diverges by root test.
(e) converges absolutely.

Answer Key :

Question	Answer
18 (092)	A
28 (092)	A
13 (091)	A
19(091)	A
21(091)	A
21(083)	B
25(083)	C
16 (082)	A
18(082)	A
19 (081)	D
3 (073)	A
18 (073)	A
20 (073)	A
25(072)	D
26(072)	A
24 (071)	A
19 (063)	A
20(063)	--
25 (063)	--
15 (062)	B
19 (062)	C

8(061)	E

Old Exam 11.8 :

(092)

13. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} 2^n n^p (x+1)^n, \text{ where } p \text{ is a constant,}$$

is

(a) $\frac{1}{2}$

(b) 2

(c) 1

(d) 4

(e) $\frac{1}{4}$

19. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$ is

(a) $\left[-\frac{1}{2}, 0\right]$

(b) $\left[\frac{1}{4}, 1\right]$

(c) $\left(-\frac{1}{2}, 0\right)$

(d) $\left(\frac{1}{4}, 1\right]$

(e) $\left[-\frac{1}{2}, 0\right)$

(091)

10. The interval of convergence I and the radius of convergence R of the power series

$$\sum_{n=1}^{+\infty} \frac{(x-1)^n}{n \cdot 2^n}$$

are

(a) $I = [-1, 3)$ and $R = 2$

(b) $I = (-1, 3)$ and $R = 2$

(c) $I = (-1, 1)$ and $R = 1$

(d) $I = [-2, 2]$ and $R = 2$

(e) $I = [-1, 3]$ and $R = 2$

(083)

20. The interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$$

is given by

(a) $(-\infty, \infty)$

(b) $\left[\frac{5}{2}, \frac{7}{2}\right)$

(c) $\left[\frac{5}{2}, \frac{7}{2}\right]$

(d) $\left(\frac{5}{2}, \frac{7}{2}\right)$

(e) $(2, 4)$

(082)

6. The radius of convergence of the power series $\sum_{n=0}^{+\infty} \frac{(x-1)^n}{3^n}$ is

(a) $R = 3$

(b) $R = 1$

(c) $R = \frac{1}{3}$

(d) $R = \infty$

(e) $R = 0$

17. The interval of convergence of the power series $\sum_{n=1}^{+\infty} \frac{(2x-3)^n}{n4^{2n}}$ is

(a) $\left[-\frac{13}{2}, \frac{19}{2}\right)$

(b) $\left(-\frac{13}{2}, \frac{19}{2}\right)$

(c) $\left(-\frac{13}{2}, \frac{19}{2}\right]$

(d) $\left[-\frac{13}{2}, \frac{19}{2}\right]$

(e) $\left[\frac{13}{2}, \frac{19}{2}\right)$

(081)

23. The interval of convergence I and the radius of convergence R of the power series $\sum_{n=1}^{+\infty}(-1)^n\frac{x^n}{4^n\cdot n^3}$ are

(a) $I=[-3,3)\ ,\quad R=3$

(b) $I=(-4,4]\ ,\quad R=4$

(c) $I=(3,4)\ ,\quad R=\frac{1}{2}$

(d) $I=[-4,4]\ ,\quad R=4$

(e) $I=(-4,4)\ ,\quad R=4$

24. If the power series $\sum_{n=0}^{+\infty} c_n(x+2)^n$ has a radius of convergence $R = 3$, then which one of the following is **TRUE**?

(a) $\sum_{n=0}^{+\infty} \frac{c_n}{2^n}$ is divergent

(b) $\sum_{n=0}^{+\infty} (-1)^n c_n 5^n$ is convergent

(c) $\sum_{n=0}^{+\infty} c_n 2^n$ is divergent

(d) $\sum_{n=0}^{+\infty} c_n 4^n$ is convergent

(e) $\sum_{n=0}^{+\infty} c_n$ is convergent

12. The radius of convergence R and the interval of convergence I of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n(2x-3)^n}{n \cdot 4^n}$ are given by

(a) $R = 2, \quad I = \left(-\frac{1}{2}, \ \frac{7}{2}\right]$

(b) $R = 2, \quad I = [-4, \ 4]$

(c) $R = 2, \quad I = \left[-\frac{1}{2}, \ \frac{7}{2}\right)$

(d) $R = 4, \quad I = (-4, \ 4]$

(e) $R = 4, \quad I = \left[-\frac{1}{2}, \ \frac{7}{2}\right)$

(072)

28. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^3 2^n}$

(a) $0 < x < 3$

(b) $0 \leq x < 4$

(c) $0 \leq x \leq 4$

(d) $-\infty < x < \infty$

(e) $0 < x \leq 4$

(071)

25. The interval of convergence and the radius of convergence R of the power series $\sum_{n=0}^{+\infty} \frac{(-3)^{n+1}(2x+1)^n}{\sqrt{n+1}}$ are

(a) $\left(\frac{-2}{3}, \frac{-1}{3}\right]$; $R = \frac{1}{6}$

(b) $\left(\frac{-2}{3}, \frac{-1}{3}\right)$; $R = \frac{2}{9}$

(c) $\left[\frac{-2}{3}, \frac{1}{3}\right]$; $R = \frac{1}{6}$

(d) $\left(\frac{-2}{3}, \frac{-1}{3}\right]$; $R = \frac{1}{9}$

(e) $\left(\frac{-2}{3}, \frac{1}{3}\right]$; $R = \frac{1}{6}$

(063)

22. The interval of convergence of the power series $\sum_{n=2}^{\infty} \frac{x^{n+1}}{2n+1}$ is

(a) $(-1, 1)$
(b) $(-1, 1]$
(c) $[-1, 1)$
(d) $[-1, 1]$
(e) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(062)

16. The radius and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}$ are respectively

(a) $\frac{1}{2}$ and $\left(\frac{5}{2}, \frac{7}{2}\right)$

(b) 1 and $[2, 4)$

(c) $\frac{1}{2}$ and $\left[\frac{5}{2}, \frac{7}{2}\right)$

(d) 1 and $(2, 4)$

(e) $\frac{1}{2}$ and $\left(\frac{5}{2}, \frac{7}{2}\right]$

21. The value of a for which the series $\sum_{n=0}^{\infty} 4^n(3+a)^{-n}$ converges to 2 is equal to

(a) 5

(b) 1

(c) 3

(d) 0

(e) 6

(061)

13. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^3}$ is

(a) $[0, 1/2]$
(b) $[0, 1]$
(c) $(0, 1)$
(d) $(-\infty, +\infty)$
(e) $\{0\}$

Answer Key :

Question	Answer
13 (092)	A
19(092)	A
10 (091)	A
20 (083)	B
6 (082)	A
17 (082)	A
23 (081)	D
24(081)	E
12 (073)	A
28 (072)	C
25 (071)	A
22 (063)	--
16 (062)	C

21 (062)	A
13(061)	B

Old Exam 11.9 :

(092)

20. The power series representation for the function $f(x) = \dfrac{x}{9+x^2}$ is

(a) $\frac{1}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x}{3}\right)^{2n+1}$

(b) $\sum_{n=0}^{\infty}(-1)^n\left(\frac{x}{3}\right)^{2n}$

(c) $\frac{1}{3}\sum_{n=0}^{\infty}(-1)^{n-1}\left(\frac{x}{3}\right)^{2n+1}$

(d) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{9^n}$

(e) $\frac{1}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x}{9}\right)^{2n+1}$

(091)

27. A power series representation for $f(x) = \dfrac{2}{(1-2x)^2}$ is given by

(a) $\sum_{n=1}^{+\infty} n \cdot 2^n x^{n-1}$

(b) $\sum_{n=0}^{+\infty} 2^n \dfrac{x^{n+1}}{n+1}$

(c) $\sum_{n=0}^{+\infty} 2^n x^n$

(d) $\sum_{n=0}^{+\infty} n \cdot 2^n x^{n+1}$

(e) $\sum_{n=1}^{+\infty} \dfrac{2^n}{n} x^n$

23. $\displaystyle\int \frac{e^{-2x}-1}{x}\,dx =$

(a) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n 2^n x^n}{n \cdot n!} + C$

(b) $\displaystyle\sum_{n=1}^{+\infty} \frac{2^{n-1} x^{n-1}}{(n-1)!} + C$

(c) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n 2^{n-1} x^{n-1}}{(n-1)!} + C$

(d) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n 2^n x^{n+1}}{(n+1)!} + C$

(e) $\displaystyle\sum_{n=1}^{+\infty} \frac{2^n x^n}{(n+1)!} + C$

24. For $|x| < 1$, $\displaystyle\sum_{n=2}^{+\infty} n(n-1)x^{n-2} =$

(a) $\dfrac{2}{(1-x)^3}$

(b) $\dfrac{2x}{(1-x)^3}$

(c) $\dfrac{x}{1-x}$

(d) $\dfrac{1}{(1-x)^2}$

(e) $\dfrac{3}{(1-x)^2}$

(082)

28. The power series representation for the function $f(x) = \dfrac{x}{(x-2)^2}$ is (Hint: You may use differentiation)

(a) $\sum_{n=0}^{\infty} \dfrac{n+1}{2^{n+2}} x^{n+1}$

(b) $\sum_{n=0}^{\infty} \dfrac{n+1}{2^{n+2}} x^{n}$

(c) $\sum_{n=0}^{\infty} \dfrac{n+1}{2^{n+1}} x^{n+1}$

(d) $\sum_{n=0}^{\infty} \dfrac{n}{2^{n}} x^{n}$

(e) $\sum_{n=0}^{\infty} \dfrac{n+1}{2} x^{n+1}$

(081)

28. The power series representation of $f(x) = \dfrac{x^2}{4+x^3}$ is

(a) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+3}}{4^{n+1}}\,, \quad |x| < \sqrt[3]{4}$

(b) $\displaystyle\sum_{n=0}^{+\infty} \frac{x^{3n+2}}{4^n}\,, \quad |x| < \sqrt[3]{4}$

(c) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \left(\frac{x}{4}\right)^{3n+2}\,, \quad |x| < 4$

(d) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{x^{3n+2}}{4^{n+1}}\,, \quad |x| < \sqrt[3]{4}$

(e) $\displaystyle\sum_{n=0}^{+\infty} \frac{x^{n+3}}{4^{n+1}}\,, \quad |x| < 4$

(073)

23. Using the power series of $\displaystyle\int \frac{dx}{1+x^2}$, then the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+3}$ is equal to

(a) $\dfrac{\pi}{4} - \dfrac{2}{3}$

(b) $\dfrac{\pi}{4} - \dfrac{1}{3}$

(c) $\dfrac{\pi}{4} + \dfrac{2}{3}$

(d) $\dfrac{\pi}{4} + \dfrac{1}{3}$

(e) $\dfrac{\pi}{4} + \dfrac{4}{3}$

(072)

23. Using the power series of $\ln(1-x)$, the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n3^n}$ is equal to

(a) $\ln 3$

(b) 1

(c) $\ln \frac{3}{2}$

(d) $\ln \frac{2}{3}$

(e) $\ln 2$

(071)

26. The value of the integral $\int_0^{1/3} \frac{x^2}{1+x^7}\,dx$ is equal to

(a) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+3)\cdot 3^{7n+3}}$

(b) $\sum_{n=0}^{+\infty} \frac{(-1)^n \cdot 3^{7n+3}}{7n+3}$

(c) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1)\cdot 3^{7n+1}}$

(d) $\sum_{n=0}^{+\infty} \frac{1}{(7n+1)\cdot 3^{7n+3}}$

(e) $\sum_{n=1}^{+\infty} \frac{(-1)^n(7n+3)}{3^{7n+1}}$

(063)

24. $\frac{x}{1+2x} =$

(a) $\sum_{n=0}^{\infty} (-1)^n 2^n x^{n+1}$

(b) $\sum_{n=0}^{\infty} 2^n x^{n+1}$

(c) $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$

(d) $\sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{2}\right)^n$

(e) $\sum_{n=0}^{\infty} x(1+2x)^n$

(062)

25. A power series representation for $f(x) = \dfrac{3x^3}{(x-3)^2}$ is given by

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{n+3}}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^{n+2}} x^n$

(c) $\displaystyle\sum_{n=1}^{\infty} n \left(\frac{x}{3}\right)^n$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{3^n} x^n$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n} x^{n+2}$

Answer Key :

Question	Answer
20 (092)	A
27 (091)	A
23 (083)	A
24(083)	A
28 (082)	A
28 (081)	D
23 (073)	A
23 (072)	C
26 (071)	A
24 (063)	--
25 (062)	E

Old Exam 11.10 :

(092)

21. The first three nonzero terms of the Taylor series of $f(x) = \sin(2x)$ about $a = \dfrac{\pi}{2}$ are given by

(a) $-2\left(x - \frac{\pi}{2}\right) + \frac{4}{3}\left(x - \frac{\pi}{2}\right)^3 - \frac{4}{15}\left(x - \frac{\pi}{2}\right)^5$

(b) $1 - 2\left(x - \frac{\pi}{2}\right) + \frac{4}{3}\left(x - \frac{\pi}{2}\right)^3$

(c) $-2\left(x - \frac{\pi}{2}\right) + \frac{4}{3}\left(x - \frac{\pi}{2}\right)^2 - \frac{4}{15}\left(x - \frac{\pi}{2}\right)^3$

(d) $2\left(x - \frac{\pi}{2}\right) + \frac{4}{3}\left(x - \frac{\pi}{2}\right)^3 - \frac{4}{15}\left(x - \frac{\pi}{2}\right)^5$

(e) $-2\left(x - \frac{\pi}{2}\right) + 4\left(x - \frac{\pi}{2}\right)^2 - 4\left(x - \frac{\pi}{2}\right)^5$

22. If the Maclaurin series of $(1 + x)^{3/2}$ is

$$A + Bx + Cx^2 + Dx^3 + Ex^4 + \cdots,$$

then $D + E =$

(a) $-\dfrac{5}{128}$

(b) $\dfrac{9}{128}$

(c) $\dfrac{7}{16}$

(d) $-\dfrac{7}{16}$

(e) $-\dfrac{7}{128}$

(091)

5. The sum of the series $\sum_{n=0}^{+\infty} \frac{1}{3^n \cdot n!}$ is equal to

(a) $\sqrt[3]{e}$

(b) e^3

(c) $\sin(3)$

(d) $\sqrt[3]{e} - 1$

(e) e^{-3}

7. The coefficient of x^{10} in the Maclaurin series of $f(x) = \sin(x^2)$ is equal to

(a) $\dfrac{1}{120}$

(b) 0

(c) $\dfrac{-1}{6}$

(d) $\dfrac{1}{6}$

(e) $\dfrac{1}{10}$

14. The first three terms of the Taylor series of $f(x) = \cos(2x)$ about $a = \pi$ are given by

(a) $1 - 2(x-\pi)^2 + \dfrac{2}{3}(x-\pi)^4$

(b) $1 - 2(x-\pi) - 2(x-\pi)^2$

(c) $1 - 2(x-\pi)^2 + 16(x-\pi)^4$

(d) $-1 + 2(x-\pi) + \dfrac{4}{3}(x-\pi)^3$

(e) $1 + 2(x+\pi)^2 - \dfrac{2}{3}(x+\pi)^4$

(083)

7. The first four terms of the Taylor series of $f(x) = \dfrac{1}{\sqrt{x}}$ about $a = 1$ are given by

(a) $1 - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 - \frac{5}{16}(x-1)^3$

(b) $1 - (x-1) + (x-1)^2 + (x-1)^3$

(c) $1 + \frac{1}{2}(x-1) - \frac{3}{8}(x-1)^2 - \frac{2}{3}(x-1)^3$

(d) $1 - \frac{1}{2}(x-1) + \frac{3}{4}(x-1)^2 - \frac{15}{8}(x-1)^3$

(e) $\frac{1}{2} - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 - \frac{15}{7}(x-1)^3$

12. The sum of the series

$$-\frac{2^4}{4!}+\frac{2^6}{6!}-\frac{2^8}{8!}+\frac{2^{10}}{10!}-\cdots$$

is equal to

(a) $-2-\sin 2$

(b) $1-\cos 2$

(c) $-\frac{1}{2}-2\cos 2$

(d) $2-\cos 2$

(e) $-1-\cos 2$

19. The Taylor series of $f(x) = \dfrac{1}{x}$ about $x = 2$ is

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^n$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x-2)^n$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^{n+1}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x+2)^n$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x+2)^n$

20. The coefficient of x^4 in the Maclaurin series of $\cos^2 x$ is

(a) $\dfrac{1}{3}$

(b) $\dfrac{2}{3}$

(c) 2

(d) $\dfrac{1}{2}$

(e) $\dfrac{1}{4}$

23. The sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!}$ is

(a) $\frac{1}{\sqrt{2}}$

(b) $\sqrt{2}$

(c) $\frac{1}{2}$

(d) $\frac{\sqrt{3}}{2}$

(e) $\sqrt{3}$

(081)

4. The first four terms of the Taylor series of $f(x) = 4 + \ln x$ about $a = 1$ are given by

(a) $4 + x - \frac{1}{2}\,x^2 + \frac{1}{3}\,x^3$

(b) $4 + (x+1) - (x+1)^2 + 2(x+1)^3$

(c) $4 + (x-1) - \frac{1}{2}\,(x-1)^2 + \frac{1}{3}\,(x-1)^3$

(d) $4 + 5(x-1) - \frac{3}{2}\,(x-1)^2 + (x-1)^3$

(e) $4 + (x-1) - (x-1)^2 + 2(x-1)^3$

22. The sum of the series $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{2^{4n+1}(2n)!}$

(a) is equal to $\frac{1}{2\sqrt{2}}$

(b) is equal to $\frac{1}{\sqrt{2}}$

(c) is equal to -1

(d) is equal to $\sqrt{2}$

(e) does not exist

27. The Maclaurin series for $f(x) = e^{-x^2/3}$ is given by

(a) $\displaystyle\sum_{n=0}^{+\infty} \frac{x^{2n}}{3^n \cdot n!}$

(b) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{3 \cdot n!}$

(c) $\displaystyle\sum_{n=1}^{+\infty} (-1)^n \frac{x^{n}}{3^n \cdot n!}$

(d) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{3^n \cdot n!}$

(e) $\displaystyle\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{x^{2n}}{9^n \cdot n!}$

14. If $ax + bx^2 + cx^3$ is the sum of the first three terms of the Maclarium series of $e^{2x} \sin x$, then $a + b + c =$

(a) $\dfrac{29}{6}$

(b) $\dfrac{7}{3}$

(c) $\dfrac{5}{6}$

(d) $\dfrac{14}{3}$

(e) $\dfrac{31}{6}$

(072)

27. The first 5 terms of the Taylor series of the function $f(x) = x\ln x$ at $x = 1$ are

(a) $(x-1) + \dfrac{(x-1)^2}{2} - \dfrac{(x-1)^3}{6} + \dfrac{(x-1)^4}{12} - \dfrac{(x-1)^5}{20}$

(b) $(x-1) + \dfrac{(x-1)^2}{2} + \dfrac{(x-1)^3}{6} + \dfrac{(x-1)^4}{12} + \dfrac{(x-1)^5}{20}$

(c) $(x-1) + \dfrac{(x-1)^2}{2!} - \dfrac{(x-1)^3}{3!} + \dfrac{(x-1)^4}{4!} - \dfrac{(x-1)^5}{5!}$

(d) $(x-1) - \dfrac{(x-1)^2}{2} + \dfrac{(x-1)^3}{6} - \dfrac{(x-1)^4}{12} + \dfrac{(x-1)^5}{20}$

(e) $(x-1) + \dfrac{(x-1)^2}{2!} + \dfrac{(x-1)^3}{3!} + \dfrac{(x-1)^4}{4!} + \dfrac{(x-1)^5}{5!}$

(071)

4. The sum of the series $1 - \ln 3 + \dfrac{(\ln 3)^2}{2!} - \dfrac{(\ln 3)^3}{3!} + \cdots$

(a) is equal to $\dfrac{1}{3}$

(b) is equal to 3

(c) does not exist

(d) is equal to $e^{1/3}$

(e) is equal to e^3

28. The Maclaurin series of $f(x) = x\cos(x^3)$ is

(a) $\sum_{n=0}^{+\infty} (-1)^n \frac{x^{6n+1}}{(2n)!}$

(b) $\sum_{n=0}^{+\infty} \frac{x^{6n}}{(2n)!}$

(c) $\sum_{n=0}^{+\infty} (-1)^n \frac{x^{6n+1}}{(6n+1)!}$

(d) $\sum_{n=0}^{+\infty} (-1)^n \frac{x^{3n+1}}{(2n)!}$

(e) $\sum_{n=0}^{+\infty} (-1)^n \frac{x^{5n+1}}{(2n)!}$

(062)

23. For $x > 0$, the series $\sum_{n=0}^{\infty} \frac{(-2)^n (\ln x)^n}{n!}$ converges to

[Hint: Use the Maclaurin series of e^x]

(a) x

(b) e^x

(c) $\frac{1}{x}$

(d) $\frac{1}{x^2}$

(e) x^2

(061)

14. If the first three nonzero terms of the Maclaurin series for $\tan^{-1} x$ are used, then the approximation of $\tan^{-1} 1$ is:

(a) $13/15$
(b) $11/15$
(c) $2/5$
(d) $23/5$
(e) $14/15$

15. The Maclaurin series for the function $f(x) = \frac{1-\cos x}{x^2}$, $x \neq 0$ is

(a) $\sum\limits_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+2)!}$
(b) $\sum\limits_{n=0}^{\infty} \frac{x^{2n}}{(2n+2)!}$
(c) $\sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+2)!}$
(d) $\sum\limits_{n=1}^{\infty} \frac{x^{2n+3}}{(2n)!}$
(e) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$

16. For $x \neq 0$, the sum of the series $\sum\limits_{n=0}^{\infty} \frac{x^n}{2^n (n+1)!}$ is equal to

(a) $e^{\frac{x}{2}} - 1 - \frac{x^2}{4}$
(b) $\frac{1}{2} \sin x$
(c) $\frac{x}{2} \cos x$
(d) $\frac{2e^x}{x}$
(e) $\frac{2}{x}(e^{\frac{x}{2}} - 1)$

Answer Key :

Question	Answer
21 (092)	A
22(092)	A
5 (091)	A
7 (091)	A
7 (083)	A
12 (083)	E
19 (082)	A
20 (082)	A
23 (082)	A
4 (081)	C
22 (081)	A
27 (081)	D

14 (073)	A
27 (072)	A
4 (071)	A
28 (071)	A
23 (063)	--
23 (062)	D
14 (061)	A
15 (061)	C
16 (061)	E

Old Exam 10.1 :

(093)

a) Find the rectangular (Cartesian) equation of the parametric curve given by

$$\mathbf{x} = \mathbf{t} - \frac{\sqrt[3]{\mathbf{t}}}{2}, \quad \text{and} \quad \mathbf{y} = \mathbf{t} + \frac{\sqrt[3]{\mathbf{t}}}{2}$$

b) A parametric curve is given by the equations

$$\mathbf{x} = \ln \mathbf{t} \quad \text{and} \quad \mathbf{y} = \sqrt{\mathbf{t}}, \qquad \mathbf{1} \le \mathbf{t} \le \mathbf{e}^2.$$

Sketch the curve and indicate with an arrow the direction in which it is traced.

(091)

b) A parametric curve is given by the equations

$$x = 2\cos t - 1 \qquad \text{and} \qquad y = 1 + \cos t.$$

Sketch the curve and indicate with an arrow the direction in which it is traced as the parameter increases from 0 to π.

(082)

Q.1 a) i) Find the Cartesian equation of the curve whose parametric equations are given by $x = 2\cot t$, $y = 2\sin^2 t$.

ii) Find the point where the curve intersects the y-axis.

(081)

Q.7: (6 pts) The graph of the curve represented by $x = 4\cos\theta$, and $y = 5\sin\theta$, is:

Q.11: (6 pts) The Cartesian equation for the parametric equations $x = 9\sec t$ and $y = 8\tan t$ is:

Old Exam 10.2 :

(093)

a) A curve C is defined by the parametric equations

$$x = t - \sin t \quad \text{and} \quad y = 1 - \cos t, \qquad 0 \leq t \leq 2\pi.$$

Find (if exist) the points on C where the tangent is horizontal or vertical.

b) For the curve given by

$$x = 2\cos\theta \quad \text{and} \quad y = 2\sin\theta$$

find the slope and concavity at the point $(\sqrt{2}, \sqrt{2})$.

(092)

1. (a) Find the length of the curve

$$x = a(\cos\theta + \theta\sin\theta), \; y = a(\sin\theta - \theta\cos\theta); \quad 0 \leq \theta \leq \pi \quad (a > 0).$$

(b) A curve is defined by the parametric equations

$$x = t - e^t, \quad y = t + e^{-t}.$$

(i) Find $\dfrac{d^2y}{dx^2}$.

(ii) For which value of t is the curve concave upward?

(091)

a) A curve C is defined by the parametric equations

$$x = \theta^2 \qquad \text{and} \qquad y = 2(1 - \cos\theta), \qquad 0 \leq \theta \leq 2\pi.$$

Find (if exist) the points on C where the tangent is horizontal or vertical.

b) For the curve given by

$$x = \sqrt{t} \qquad \text{and} \qquad y = \frac{1}{2}(t^2 - 2), \qquad t \geq 0.$$

Find the slope and concavity at the point $(\sqrt{2}, 1)$.

b) Find the length of the parametric curve

$$x = \sin t - t\cos t \qquad \text{and} \qquad y = \cos t + t\sin t, \qquad 0 \leq t \leq \pi.$$

(083)

b) Find the area of the surface obtained by rotating the curve with parametric equations $x = a\cos^3\theta,\ y = a\sin^3\theta,\ 0 \leq \theta \leq \pi/2$ about the x-axis.

(082)

b) Find the points on the curve $x = 6t - t^3$, $y = 3t^2$ where the tangent is parallel to the line with equation $y = 5 - 2x$.

(081)

Q.1: (12 pts) Find equations of all tangent lines to the parametric curve given by $x = t^5 - 4t^3$, $y = t^2$, at $(0, 4)$.

(073)

1. At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent parallel to the line with equations $x = -7t$, $y = 12t - 5$? (6 marks)

(072)

1. (a) At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent parallel to the line with equation $x = -7t$, $y = 12t - 5$? (4 marks)

   (b) Find the points on the curve $r = \cos\theta + \sin\theta$ where the tangent line is horizontal or vertical.

   (4 marks)

(063)

1. Given the parametric curve $x = t - e^{2t}$, $y = t + e^{2t}$

   (a) Find $\dfrac{dy}{dx}$.

   (b) For which value of t the parametric curve has a vertical tangent line.

(062)

1. Consider the parametric curve with equations

$$\begin{aligned} x &= t^2 + 1 \\ y &= 3t - t^3. \end{aligned}$$

   (a) Find the points of intersection with this curve with (a) the x-axis, (b) the y-axis.

   (b) Find points on the curve where the tangent line is

      (i) horizontal

      (ii) vertical.

(061)

1. [5pts]For the parametric curve $x = 10 - t^2$, $y = t^3 - 4t$, find the points where the tangent line is (a) horizontal (b) vertical. Find also $\dfrac{d^2y}{dx^2}$.

Old Exam 10.3 :

(093)

a) Sketch the curve with polar equation

$$r = 2(1 - \sin\theta).$$

b) Find the equation of the tangent line to the polar curve in part a) at $\theta = \pi$.

(092)

2. (a) Find equation of the tangent line of the curve with polar equation

$$r = 5 - 4\sin\theta \text{ at } \theta = \pi.$$

(b) Sketch the polar curve $r = |\sin 2\theta|, \quad 0 \le \theta \le \pi$.

(091)

a) Find the rectangular (Cartesian) equation for the polar curve given by

$$r = \sin^2\frac{\theta}{2} + \tan\theta$$

a) Sketch the curve with polar equation

$$r = 2\cos\left(\frac{\theta}{2}\right), \qquad 0 \le \theta \le 2\pi\ .$$

b) Find the equation of the tangent line to the polar curve in part a) at $\theta = \pi/2$

(083)

Q.1 a) Find a Cartesian equation of the curve whose polar equation is given by $r = \dfrac{\sin 2\theta - \cos 2\theta}{\sin\theta\cos\theta}$

Q.2 Consider the polar equation $r = 1 - \sin\theta$.

i) Sketch the curve of the given polar equation

ii) Find the equation of the tangent line to the polar curve at $\theta = \pi/3$

(082)

Q.2 Consider the polar equation $r = 2 + 2\cos 2\theta$.

i) Sketch the curve of the given polar equation

ii) Find the slope of the tangent line to the polar curve at $\theta = \pi/4$

(081)

Q.2: (14 pts) Consider the polar curve $C: \ r = f(\theta) = \cos(2\theta)$, $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

(a) Sketch the curve.
(b) Setup the integral for the area enclosed by the curve.
(c) Setup the integral for the arc length of curve.

Q.6: (8 pts) Find the Cartesian equation of the curve whose polar equation is given by $r = \sec(\theta) - \csc(\theta)$.

Q.8: (6 pts) What does the polar equation $r = \tan\theta \sec\theta$ represents ?

(073)

2. Find the points on the cardioid $r = 1 + \sin\theta$ where the tangent is horizontal or vertical. (6 marks)

(072)

(b) Find the points on the curve $r = \cos\theta + \sin\theta$ where the tangent line is horizontal or vertical.

(071)

Q1. **Plot** the points whose polar coordinates are given in parts (a)-(c) below.

(a): $(3, \pi/4)$

(b): $(-3, \pi/6)$ (c): $(-3, -\pi/6)$

(d): **Give** and **plot** **<u>two</u>** other representations of the point $(4, 2\pi/3)$.

Q2(a). Find polar coordinates of the point $(-2, 2\sqrt{3})$

Q2(b). Find Cartesian coordinates of the point $(6, 3\pi/4)$

Q2(c). Sketch the polar curve $r = 1 + 2\cos\theta$

Q3(a). Given the curve $r = \sin\theta$, find slope of the tangent line at $\theta = \pi/4$

Q3(b). For the curve $r = \sin\theta$, find points at which there is a horizontal tangent line, vertical tangent line or a singular point. **Points (1)**

(063)

3. Find the slope of the tangent line to the polar curve $r = 4 - 3\sin\theta$ at $\theta = \pi$.

(061)

2. [5pts]Find the slope of the tangent line to the polar curve $r = \dfrac{1}{2 + 2\cos\theta}$ at the point with $\theta = \pi/2$.

Old Exam 10.4 :

(093)

Find the area common to the circles

$$x^2+y^2=4 \quad \text{and} \quad x^2+y^2=4x$$

Find the Arclength of the polar curve given by

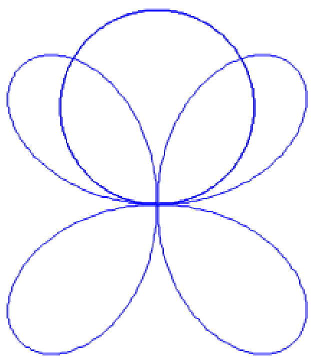
$$r=\sin^3\left(\frac{\theta}{3}\right), \qquad 0\le\theta\le\pi.$$

(092)

3. Find the area outside the curve $r=\dfrac{1}{2}$ and inside the curve $r=\cos 2\theta$ $\left(-\dfrac{\pi}{2}\le\theta\le\dfrac{\pi}{2}\right)$.

(091)

a) The graphs of the four-leaf rose curve $r = \sin 2\theta$ and the circle $r = \sin\theta$ are drawn below for you.
Find the area of the shaded region inside both curves as a sum of two definite integrals. (**DO NOT CALCULATE** the integrals)



(083)

Q.3 Compute the area that lies inside both curves $r=2\cos 2\theta$ and $r=1$.

(082)

Q.3 a) Find the area of the region inside the polar curve $r = 2 + 2\sin\theta$ and outside the curve $r = 3$.

(081)

Q.3: Find the area inside the curve $r = 1 + \sin(\theta)$ and outside the curve $r = \sin(\theta)$ when $\frac{\pi}{3} \le \theta \le \frac{\pi}{2}$.

(073)

3. Find the area of the region enclosed by the inner loop of $r = 1 + 2\sin\theta$. Also draw the graph of the curve. (6 marks)

OR

Find the area of the region that lies inside both the curves $r^2 = 2\sin 2\theta$ and $r = 1$. Also draw the graph of these curves. (6 marks)

(072)

2. Find the area of the region that lies inside the circle $r = 3\sin\theta$ and out side the cardioid $r = 1+\sin\theta$. Also draw the graph of these curves. (6 marks)

OR

Find the area between a large loop and enclosed small loop of the curve $r = 1 + 2\cos\theta$. Also draw the graph of this curve. (6 marks)

(071)

Q4. **Plot** the curve $r = \cos 3\theta$ and **find** area of the entire region swept by it.

Q5. **Set up** integral to find area of the region that lies inside both the curves given by $r = 2\sin\theta$ and $r = 2\cos\theta$

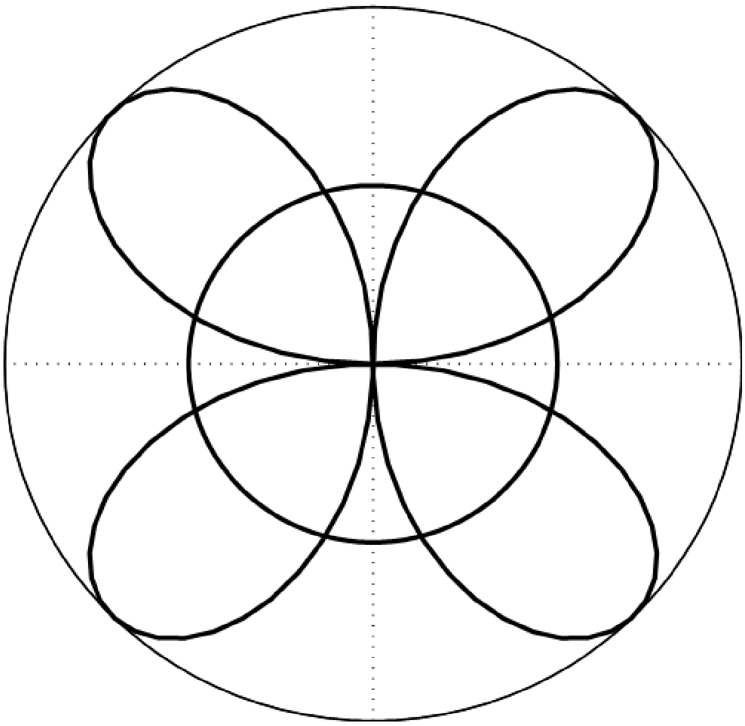
(063)

4. Set up an integral to find the arc length of the rose $r = \cos 2\theta$. (Do not evaluate the integral)

5. Find the area of the region inside the circle $r = 3\cos\theta$ and outside the cardioid $r = 1 + \cos\theta$.

(062)

2. Consider the polar curve $r = 2\sin 2\theta$. A sketch is drawn for you.



Write as an integral the area of the region in the first quadrant outside the circle $r = 1$ and inside the curve with polar equation $r = 2\sin 2\theta$.

(061)

3. [5pts]Find the area of the region inside the cardioid $r = 2 + 2\cos\theta$ and to the right of the line $r\cos\theta = 3/2$.

Review Chapter 10

Review Chapter 10 :

(092)

1. The cartesian equation for the curve $x = 3 + \cos t$, $y = 2 + \sin t$ is:

(a) $x^2 + y^2 - 6x - 4y = -12$

(b) $(x-3)^2 + (y-2)^2 = -1$

(c) $(x-3)^2 + (y-2)^2 = 4$

(d) $x^2 - y^2 - 6x + 4y = -4$

(e) $x^2 - y^2 = 4$

2. Find the area of the region bounded by $r = 1 + \sin\theta$ for $\dfrac{\pi}{4} \le \theta \le \pi$.

(a) $\dfrac{9\pi}{16} + \dfrac{\sqrt{2}}{2} + \dfrac{9}{8}$

(b) $\dfrac{9\pi}{16} + \dfrac{\sqrt{2}}{2}$

(c) $\dfrac{9\pi}{4}$

(d) $\dfrac{2\pi}{3}$

(e) $\dfrac{2\pi}{6}$

(091)

(2) The equation of the tangent line to the polar curve

$$r = \cot\theta \quad \text{at} \quad \theta = \frac{\pi}{6} \quad \text{is}$$

(A) $y = \frac{\sqrt{3}}{15}x + \frac{2\sqrt{3}}{5}$

(B) $y = x + \sqrt{3}$

(C) $y = \frac{\sqrt{3}}{5}x + \frac{2\sqrt{3}}{3}$

(D) $y = -2x + 5$

(E) $y = \sqrt{3}$

(083)

1. The curve $C:\ x = t^3 - 12t,\ y = t^2$ is concave upward on the interval

   (A) $-2 < t < 2$

   (B) $t > 2$

   (C) $t < -2$

   (D) $-2 \le t \le 2$

   (E) $t < 0$

2. The length of the curve $x = e^t + e^{-t},\ y = 5 - 2t,\quad 0 \le t \le 3$ is equal to

   (A) $e^3 - e^{-3}$

   (B) $e^3 + e^{-3}$

   (C) $e^3 - e^{-3} - 2$

   (D) $e^3 + e^{-3} + 2$

   (E) $e^3 + e^{-3} - 2$

3. The area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$ is

(A) π

(B) 2π

(C) $\dfrac{\pi}{2}$

(D) $\pi - 2$

(E) $\pi + 2$

(082)

1. The positions of two particles P_1 and P_2 at time t $(0 \le t \le 2\pi)$ are given by:

$$P_1 : x_1 = 3\sin t, \quad y_1 = 2\cos t$$

$$P_2 : x_2 = -3 + \cos t, \quad y_2 = 1 + \sin t.$$

These particles meet at the point

(A) $(-3, 0)$

(B) $(0, 2)$

(C) $(-1, 2)$

(D) $(0, \frac{1}{2})$

(E) $(-1, -3)$

2. The polar equation $r = \dfrac{6}{3\cos\theta + 2\sin\theta}$ represents a

(A) line

(B) circle

(C) hyperbola

(D) parabola

(E) cardioid

(081)

1. The curve $C : x = t - \ln t,\ \ y = t + \ln t$ is concave down on

(a) $(1, \infty)$

(b) $(0, 1)$

(c) $(0, \infty)$

(d) $(-\infty, 0) \cup (1, \infty)$

(e) $(-\infty, 1)$

2. The slope of the tangent line to the polar curve $r = 1 + \sin\theta$ at $\theta = \dfrac{\pi}{4}$ is

(a) $-\dfrac{\sqrt{2}+2}{\sqrt{2}}$

(b) $-\dfrac{1}{\sqrt{2}}$

(c) $1+\dfrac{\sqrt{2}}{2}$

(d) $\dfrac{\sqrt{2}-2}{\sqrt{2}}$

(e) $\dfrac{\sqrt{2}}{2-\sqrt{2}}$

3. The area of the region that lies inside both curves $r = 4\cos\theta$ and $r = 4\sin\theta$ is

(a) $2\pi - 4$

(b) $2\pi + 4$

(c) 4π

(d) $\pi + 2$

(e) $\pi - 2$

Answer Key：

Question	Answer
1 (092)	A
2 (092)	A
2(091)	A
1 (083)	A
2 (083)	A
3 (083)	A
1 (082)	A
2 (082)	A
1 (081)	B
2 (081)	C
3 (081)	C

Old Exam 12.1 :

(093)

(i) Find the equation of the sphere centered at $(\mathbf{3, 5, -1})$ passing through the point $(\mathbf{4, 2, -3})$.
(ii) Find its intersection with the plane $\mathbf{x - y + 2 = 0}$.

(092)

4. (a) Prove that the mid-point of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

(b) Find the equation of the sphere if one of its diameters has end-points $(2, 1, 4)$ and $(4, 3, 10)$.

(082)

Q.4 A sphere has equation $x^2+y^2+z^2 = 10y-16z+C$, where C is a constant.

i) Find the center of the sphere

ii) Find the radius of the sphere in terms of C.

iii) If the radius of the sphere is equal to 10, find the points where the sphere intersects the y-axis.

(081)

Q.4: (9 pts) Find an equation of the sphere if one of its diameters has end points at $A(1, 4, -2)$ and $B(-7, 1, 2)$. What is the intersection of this sphere with the xz−plane ?

(063)

3. Consider the points P so that the distance of P from $A(0, 0, 0)$ is twice the distance of P from $B(1, 0, 0)$. Show that the set of all such points is a sphere.

Old Exam 12.2 :

(093)

Let

$$\vec{a} = \langle 1, 2, -3 \rangle , \quad \text{and} \quad \vec{b} = \langle -2, -1, -5 \rangle .$$

Find a vector $\vec{v}$ with length 3 that has the same direction as

$$\vec{a} - 2\vec{b} .$$

(091)

a) Let

$$\vec{a} = \langle -2, 2, 4 \rangle, \quad \text{and} \quad \vec{b} = \langle 3, 3, -1 \rangle.$$

Find a vector $\vec{v}$ with length 4 that has the same direction as

$$\vec{b} - \frac{1}{2}\vec{a}.$$

(081)

Q.9 : (6 pts) The vector that has the same direction as $\langle -3, 4, 1 \rangle$ but has length 5 is:

Old Exam 12.3 :

(093)

b) Consider the vectors

$$\vec{a} = \langle 2, 1, 2 \rangle, \quad \text{and} \quad \vec{b} = \langle 0, 3, 4 \rangle.$$

Find the vector projection of $\vec{a}$ onto $\vec{b}$.

(092)

(c) If the angle between two unit vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, then find the value of $|2\vec{a} - 3\vec{b}|$.

(091)

b) Consider the vectors

$$\vec{a} = \langle -3, 4, 12 \rangle, \quad \text{and} \quad \vec{b} = \langle 24, 8, 6 \rangle.$$

Calculate the cosine of the angle between $\vec{a}$ and $\vec{b}$, and find the vector projection of $\vec{a}$ on $\vec{b}$.

a) Find a unit vector that is orthogonal to both

$$\vec{u} = \vec{i} - 4\vec{j} + \vec{k} \qquad \text{and} \qquad \vec{v} = 2\vec{i} + 3\vec{j}.$$

b) Consider the vectors

$$\vec{u} = \langle 3, 2, x \rangle, \quad \text{and} \quad \vec{v} = \langle 2x, 4, x \rangle.$$

Find (if exist) the values of x such that $\vec{u}$ and $\vec{v}$ are orthogonal and the values of x such that $\vec{u}$ and $\vec{v}$ are parallel.

(083)

Q.4 Let $\vec{a} = \langle -1, -2, 2 \rangle$ and $\vec{b} = \langle 3, 0, 4 \rangle$.

i) Find the vector projection $\vec{v} = \text{proj}_{\vec{a}}\, \vec{b}$.

ii) Evaluate $\vec{v} \cdot (\vec{b} - \vec{v})$.

(082)

Q.5 Let $\vec{a} = \langle \sqrt{2}, 1, 1 \rangle$ and $\vec{b} = \langle -\sqrt{2}, 4, -1 \rangle$ be two vectors in $\mathbb{R}^3$.

i) Find the scalar projection and vector projection of $\vec{b}$ onto $\vec{a}$.

ii) Find the angle between the vectors $\vec{a}$ and $\vec{a} + \vec{b}$.

iii) If $\vec{r} = \langle x, y, z \rangle$, show that the vector equation $(\vec{r} - \vec{a}) \cdot (\vec{r} - \vec{b}) = 0$ represents a sphere.

(081)

Q.10: (6 pts) The vector projection of $u = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ onto $v = 5\hat{\imath} - \hat{\jmath} + 2\hat{k}$ is given by:

(072)

4. (a) Find two unit vectors orthogonal to both the vectors $\vec{i} - \vec{j} + \vec{k}$ and $4\vec{j} + 4\vec{k}$.

(062)

5. (a) Find the vector projection of $\overrightarrow{DE}$ on $\overrightarrow{DP}$ where $D = (0, 1, 1)$, $E = (-2, 4, 3)$ and $F = (1, 2, -1)$.

   (b) Find values of k so that the vectors $\langle -6, k, 2 \rangle$ and $\langle k, k^2, k \rangle$ are orthogonal.

6. (a) Find the angle between the diagonals of a square.

   (b) Find the cosine of the angle between a diagonal of a cube and a diagonal of one of its faces.

(061)

4. [5pts]Find all real numbers r such that the angle between $\vec{v} = \langle 1, 1, 1 \rangle$ and $\vec{w} = \langle r+1, r, r-1 \rangle$ is $\pi/3$.

Old Exam 12.4 :

(093)

a) Find the area of the triangle with vertices $\mathbf{A}(\mathbf{1},\mathbf{1},\mathbf{1})$, $\mathbf{B}(\mathbf{2},-\mathbf{3},\mathbf{2})$, and $\mathbf{C}(\mathbf{4},\mathbf{1},\mathbf{5})$.

(092)

(a) Given the plane P defined by equation $x+2y+3z=6$, we denote by A, B and C the intersection points of the plane P with the x-, y-, and z-axes. Compute the area of triangle ABC.

(b) Determine whether the points $P(1,0,1), Q(2,4,6), R(3,-1,2)$ and $S(6,2,8)$ lie on the same plane.

(083)

Q.5 Consider a triangle with vertices $A(0,0,b)$, $B(1,1,0)$ and $C(2,3,0)$.

i) Compute the area of the given triangle.

ii) Find all values of b such that the area of the triangle is equal to 8.

(082)

Q.6 Find the area of the triangle with vertices $P(1,4,6)$, $Q(-2,5,-1)$ and $R(1,-1,1)$.

(081)

Q.12: (6 pts) Let P be the parallelogram in with vertices $A=(1,-1,2)$, $B=(2,0,1)$, $C=(3,2,-1)$, $D=(2,1,0)$. The area of P is:

(073)

5. Use the scalar triple product to determine whether the points $P(1,0,1)$, $Q(2,4,6)$, $R(3,-1,2)$ and $S(6,2,8)$ lie in the same plane. (6 marks)

(072)

4. (a) Find two unit vectors orthogonal to both the vectors $\vec{i} - \vec{j} + \vec{k}$ and $4\vec{j} + 4\vec{k}$. (4 marks)

(b) Use the scalar triple product to show that the vectors $\vec{a} = \langle 2,3,1 \rangle$, $\vec{b} = \langle 1,-1,0 \rangle$ and $\vec{c} = \langle 7,3,2 \rangle$ are coplanar. (4 marks)

(071)

Q6(a). Find a unit vector that is orthogonal to $2\underset{\to}{i} + \underset{\to}{j}$ and $\underset{\to}{j} + 3\underset{\to}{k}$

Q6(b).Find volume of the a parallelepiped determined by $\langle 6,3,-1 \rangle$, $\langle 0,1,2 \rangle$ and $\langle 4,-2,5 \rangle$. **Points (1)**

(061)

5. [5pts]Consider the points $A(1,-1,2)$, $B(2,-3,0)$, $C(-1,-2,0)$, $D(2,1,-1)$.

(a) Find the area of the triangle ABC.

(b) Find the volume of the parallelepiped that has the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ as adjacent edges.

Old Exam 12.5 :

(093)

b) Find the component form of the vector $\overrightarrow{u}$ that is perpendicular to the

plane $\mathbf{x - 3y + 4z = 0}$,

and satisfying the condition $\|\overrightarrow{u}\| = 3$.

(092)

6. (a) Find the equation of the plane passing through the origin O, which is parallel to the z-axis and is perpendicular to the plane $3x - 2y + z = 4$.

(b) Find the point of the intersection (if any) of

$$L_1 : x = -6t, y = 1 + 9t, z = -3t$$
$$L_2 : x = 1 + 2s, y = 4 - 3s, z = s.$$

(083)

Q.6 Find an equation of the plane that contains the line $x = y = 3z - 1$ and the point $B(2, 3, 0)$.

(073)

6. Find an equation of the plane that passes through the points $P(0, 1, 1)$, $Q(1, 0, 1)$ and $R(1, 1, 0)$.

(6 marks)

OR

Find the parametric equations for the line of intersection of the planes $z = x + y$ and $2x - 5y - z = 1$.

(6 marks)

(072)

6. Find an equation of the plane that passes through the points $P(1,3,2)$, $Q(3,-1,6)$ and $R(5,2,0)$. (6 marks)

OR

Find an equation of the plane with X-intercept a, Y-intercept b and Z-intercept c. (6 marks)

(071)

Q7(a). Find equation for the line through point $(1,0,4)$ and perpendicular to the plane $2x+3y+5z=6$ **Points (1)**

Q7(b). Find an equation of the plane through origin that is parallel to the plane $4x-2y+7z+12=0$. **Points (1)**

Q7(c). Find symmetric equation for the line of intersection of the planes $x+y-z=2$ and $3x-4y+5z=6$ **Points (2)**

(061)

6. [5pts]Consider the lines $L_1 : x = 3+2t,\ y = 1-t,\ z = 4+t$ and $L_2 : x = -2+3t,\ y = 3-t,\ z = 2+t$. Are they parallel? Do they intersect? If they intersect find their point of intersection.

Old Exam 12.6 :

(091)

Consider the surface given by the equation

$$y^2 = 4x^2 + 9z^2$$

a) Identify the surface and find the traces in the planes $x = k$, $y = k$, and $z = k$ to sketch the graph.

(082)

Q.2 a) Consider the quadric surface $4x^2 - 2y^2 + z^2 + 8 = 0$.

i) Find the traces of the surface in the vertical planes $y = k$. (k is a constant)

ii) Identify and sketch the surface.

(081)

Q.2: Consider the surface $\frac{z}{4} = \sqrt{x^2 + y^2}$.

(a) Describe the traces along the $z-$axis (parallel to $xy-$plane).

(b) Describe the traces along the $x-$axis (parallel to $yz-$plane).

(c) Identify and sketch the surface.

(061)

3. (a) Find equation of surface of revolution by revolving the graph of the equation $y = 4x^2$ about y-axis. Give a rough sketch of the surface. (5 points)

Review Chapter 12

Review Chapter 12 :

(092)

3. If $\vec{u} = 7\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ and $\vec{v} = -8\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, then $Comp_{\vec{v}}\vec{u}$ is:

(a) $-\dfrac{17}{\sqrt{21}}$

(b) $-\dfrac{17}{\sqrt{21}}\left(-8\hat{\imath} + 4\hat{\jmath} + 2\hat{k}\right)$

(c) $-\dfrac{34}{\sqrt{69}}$

(d) $-\dfrac{34}{\sqrt{80}}$

(e) $\dfrac{-14\hat{\imath} - 54\hat{\jmath} + 52\hat{k}}{\sqrt{21}\sqrt{83}}$

4. An equation of the plane that passes through the point $(2, -1, 3)$ and contains the line $\dfrac{x+1}{-3} = \dfrac{y-2}{2} = \dfrac{z-1}{-1}$ is:

(a) $x + 3y + 3z = 8$

(b) $x - 3y + 3z = 14$

(c) $x - 2y + 3z = 8$

(d) $x - 2y + z = -4$

(e) $2x - y + z = 1$

(091)

(20) The area of the parallelogram with adjacent sides

$$\overrightarrow{u} = \langle 1, 1, -1 \rangle \quad \text{and} \quad \overrightarrow{v} = \langle 2, 1, 1 \rangle$$

is equal to

(A) $\sqrt{14}$

(B) $\sqrt{10}$

(C) 3

(D) $\sqrt{22}$

(E) 2

(083)

4. The vector projection of $\vec{b} = \langle 1, 1, 2 \rangle$ onto $\vec{a} = \langle -1, 1, 3 \rangle$ is equal to

(A) $\left\langle -\frac{6}{11}, \frac{6}{11}, \frac{18}{11} \right\rangle$

(B) $\left\langle -\frac{6}{11}, \frac{1}{11}, \frac{3}{11} \right\rangle$

(C) $\left\langle -\frac{6}{\sqrt{11}}, \frac{8}{\sqrt{11}}, \frac{18}{\sqrt{11}} \right\rangle$

(D) $\left\langle \frac{8}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \right\rangle$

(E) $\left\langle -\frac{6}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \right\rangle$

5. A vector perpendicular to the plane that passes through the points $P(1,4,6)$, $Q(-2,5,-1)$, $R(1,-1,1)$ is

(A) $\langle -8,-3,3\rangle$

(B) $\langle 6,-3,3\rangle$

(C) $\langle -8,-3,-3\rangle$

(D) $\langle -8,3,3\rangle$

(E) $\langle -6,3,3\rangle$

(082)

3. If the angle between two unit vectors $\vec{u}_1$ and $\vec{u}_2$ is $\frac{\pi}{3}$, then $|\,2\vec{u}_1-\vec{u}_2|$ is equal to

(A) $\sqrt{3}$

(B) 3

(C) 1

(D) $5-\sqrt{3}$

(E) $3-\sqrt{3}$

4. A parallelepiped is determined by the vectors $\vec{u} = \langle 0, 4, 2\rangle$, $\vec{v} = \langle 0, 4, -1\rangle$, $\vec{w} = \langle m, 1, 3\rangle$, where m is a positive real number. If the volume of the parallelepiped is 60, then m is equal to

(A) 5

(B) 10

(C) 6

(D) 12

(E) 4

(081)

4. Vector projection of $\vec{u} = \langle 1, 2, 3\rangle$ onto $\vec{v} = \langle 1, 4, 0\rangle$ is

(a) $\langle \frac{9}{17}, \frac{36}{17}, 0\rangle$

(b) $\langle \frac{9}{\sqrt{17}}, \frac{36}{\sqrt{17}}, 0\rangle$

(c) $\langle \frac{9}{14}, \frac{36}{14}, 0\rangle$

(d) $\langle \frac{9}{17}, \frac{18}{17}, \frac{27}{17}\rangle$

(e) $\langle \frac{9}{14}, \frac{18}{14}, \frac{27}{14}\rangle$

5. The value of k for which the vectors $\vec{a} = \langle 1, 4, -7 \rangle$, $\vec{b} = \langle 2, -1, 4 \rangle$ and $\vec{c} = \langle k, 0, 1 \rangle$ are coplanar is

(a) $k = 1$

(b) $k = -1$

(c) $k = -\frac{9}{23}$

(d) $k = \frac{7}{23}$

(e) $k = \frac{1}{9}$

Answer Key :

Question	Answer
3 (092)	A
4 (092)	A
20 (091)	A
4 (083)	A
5 (083)	A
3 (082)	A
4 (082)	A
4 (081)	A
5 (081)	A

Old Exam 14.1 :

(091)

b) Find and sketch the domain of the function

$$f(x,y) = \sqrt{1 + x - y^2}$$

(083)

Q.3 Let $f(x, y) = \dfrac{e^{\sqrt{x^2+y^2-1}}}{3 + \sqrt{4 - x^2 - y^2}}$.

i) Find and sketch the domain of f.

ii) Find the range of f

(082)

Q.3 a) Let $f(x, y, z) = \sqrt{16 - x^2 - y^2 - z^2}$

i) Find and describe the domain of f

ii) Find the range of f

(081)

Q.3: Let $f(x, y) = \ln\left(36 - 4x^2 - 9y^2\right)$.

(a) Find and sketch the domain of f

(b) Find the range of f

Old Exam 14.2 :

(091)

a) Consider the function

$$f(x,y) = \frac{x^2y^2}{x^2+2y^2}$$

Show that if $\lim_{(x,y)\to(0,0)} f(x,y)$ exists, it must be 0. Prove that the limit is in fact equal to 0.

b) Show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2}$$

does not exist.

(083)

Q.4 (a) Let $f(x,y) = \frac{xy^3}{x^2+y^6}$.

i) Evaluate $\lim_{(x,y)\to(0,0)} f(x,y)$ along the line $y = mx$, where m is a constant.

ii) Evaluate $\lim_{(x,y)\to(0,0)} f(x,y)$ along the curve $y^3 = x$.

iii) Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Justify your answer.

(082)

b) Let

$$f(x,y) = \begin{cases} \dfrac{3xy}{x^2+xy+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Check whether or not f is continuous at $(0,0)$.

(081)

Q.4: If $f(x,y) = \dfrac{x^2 y}{x^4 + y^2}$, does the limit $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exist? Justify your answer.

(073)

1. Determine the set of points at which the function

$$f(x,y) = \begin{cases} \frac{x^2y^3}{2x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 1, & \text{if } (x,y) = (0,0) \end{cases}$$

is continuous.

(072)

1. Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}}$ if it exists, or show that the limit does not exist.

OR

Find $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz+zx}{x^2+y^2+z^2}$ if it exists, or show that the limit does not exist.

3. Determine the set of points at which the function

$$f(x,y) = \begin{cases} \frac{x^2y^3}{2x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$$

is continuous.

(063)

2. Find $\lim_{(x,y)\to(0,0)} \dfrac{x^3 y}{2x^6 + y^2}$ along the curves $y = mx^2$, and $y = x^3$. Does the limit exist.

(062)

2. [4pts] Find $\lim_{(x,y)\to(0,0)} \dfrac{x^4 y}{x^8 + y^2}$ or show that the limit does not exist.

(061)

4. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x,y) = \begin{cases} \dfrac{\sin(2x^2+y^2)}{x^2+y^2} & (x,y) \neq (0,0) \\ 1 & (x,y) = (0,0) \end{cases}$

(5 points)

(a) Is $f(x,y)$ continuous at $(0,0)$? Give reasons.

Old Exam 14.3 :

(091)

a) Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ when $s = 3$ and $t = \frac{\pi}{4}$ for the function given by

$$w = x^2 - y^2$$

where

$$x = s\cos t, \quad y = s\sin t$$

(083)

Q.6 (a) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if the equation $z\sin x + y\cos z^2 - yz^2 + 4 = 0$ defines z as a function of two independent variables x and y.

(082)

Q.4 a) The equation $xy + xz^3 - 2yz = 5$ defines z as an implicit function of x and y. Find $\dfrac{\partial z}{\partial x}$ at the point $(3, 2, 1)$.

(081)

Q.7: Let $f(x, y) = \sin\left(\frac{x}{y}\right) + e^{\frac{z}{y}} + 3x$. Then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is equal to:

(073)

2. Find $\dfrac{\partial^6 u}{\partial x \partial y^2 \partial z^3}$ if $u = x^a y^b z^c$

3. Use Implicit Function Theorem to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $xyz = \cos(x + y + z)$

(072)

2. If $f(r, s, t) = r\ln(rs^2t^3)$, then find f_{rst} and f_{rss}.

(063)

5. Use partial derivative to find $\frac{dy}{dx}$ if $y = f(x)$ is determined implicitly by the equation $x^3 + xy^2 + y^3 = 1$.

Old Exam 14.4 :

(091)

b) Determine an equation of the tangent plane to

$$f(x,y) = 3e^{(x-y)}\ln x$$

at the point $(1,1)$.

a) Find the linear approximation of

$$f(x,y) = \frac{y}{x}$$

at $(3,6)$ and use it to approximate $f(3.03, 5.97)$.

(083)

(b) Let $z = \ln(\sqrt{x^2+y^2})$ and (x,y) changes from $(3,4)$ to $(2.95, 4.1)$. Use differentials to estimate the change Δz of z.

(082)

b) Find the linearization of $f(x,y) = \sqrt{x^2+y^2}$ at the point $(4,3)$

(081)

Q.11: If (x,y) changes from $(2,-1)$ to $(1.96,-0.95)$ in the function $z = x^2 - xy + 3y^2$, then the value of the differential dz is:

(073)

4. Find the linear approximation of the function $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ at $(2,1)$ and use it to approximate $f(1.95, 1.08)$. (6 marks)

(072)

4. Find the linear approximation of the function $f(x, y) = \sqrt{20 - x^2 - 7y^2}$ at $(2, 1)$ and use it to approximate $f(1.95, 1.08)$. (6 marks)

OR

Find an equation of the tangent plane to the surface $z = e^{x^2 - y^2}$ at $P(1, -1, 1)$. (6 marks)

(063)

1. Find the equation for the tangent plane to the graph of the equation $z = xe^{-y}$ at $P(1, 0, 1)$.

(062)

3. [4pts] Find the linear approximation of the function $f(x, y) = \ln(x - 3y)$ at $(7, 2)$ and use it to approximate $f(6.9, 2.06)$.

(061)

5. The function $f(x, y) = x^2 y$ has a local linear approximation $L(x, y) = 4y - 4x + 8$ at a point $P_0\,(x_0, y_0)$. Find the point P_0. (5 points)

Old Exam 14.5 :

(091)

a) Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ when $s = 3$ and $t = \frac{\pi}{4}$ for the function given by

$$w = x^2 - y^2$$

where

$$x = s\cos t, \quad y = s\sin t$$

(083)

Q.5 (a) Let $w = \sin(\sqrt{x} + \sqrt{y}), \quad x = \sqrt{u^2 + v^2}, \quad y = e^{uv}$.

i) Find $\dfrac{\partial w}{\partial u}$ and $\dfrac{\partial w}{\partial v}$.

ii) Evaluate $\dfrac{\partial w}{\partial u} + 2\dfrac{\partial w}{\partial v}$ at the point $(u, v) = (\pi, 0)$.

Q.6 (a) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if the equation $z\sin x + y\cos z^2 - yz^2 + 4 = 0$ defines z as a function of two independent variables x and y.

(081)

Q.9 : Find $\left.\dfrac{\partial z}{\partial x}\right|_{(\frac{\pi}{4},1,1)}$ when $x - z + 1 = \arctan(yz)$.

(073)

3. Use Implicit Function Theorem to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $xyz = \cos(x + y + z)$ (4 marks)

(072)

5. If $u = x^4y + y^2z^3$, where $x = rse^t$, $y = rs^2t^{-1}$ and $z = r^2s\sin t$, find the value of $\dfrac{\partial u}{\partial s}$ when $r = 2$, $s = 1$ and $t = 0$. Also draw a tree diagram for $\dfrac{\partial u}{\partial s}$. (6 marks)

OR

If $x - z = \tan^{-1}(yz)$, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$. (6 marks)

(062)

4. [4pts] Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{dy}$ if $yz = \ln(x - z)$.

(061)

6. (a) Let $z = \tan^{-1}\left(\dfrac{u}{v}\right)$ where $u(x, y) = 2x + y$ and $v(x, y) = 3x - y$. Find $\dfrac{\partial z}{\partial y}$. (5 points)

Old Exam 14.6 :

(091)

a) The directional derivative of $f(x,y)$ at $(1,1)$ is $\sqrt{2}$ in the direction of $\vec{u_1} = \vec{i}$, and it is -3 in the direction of $\vec{u_2} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$.
Find the directional derivative of f at $(1,1)$ in the direction of

$$\vec{u_3} = \frac{2}{\sqrt{13}}\vec{i} + \frac{3}{\sqrt{13}}\vec{j}.$$

a) Find the equation of the tangent plane and a set of parametric equations of the normal line to the surface

$$xz - yz^3 + yz^2 = 2$$

at the point $(2,-1,1)$.

b) The surface of a mountain is modeled by the equation

$$h(x,y) = 4000 - 0.001x^2 - 0.004y^2.$$

Suppose that a mountain climber is at the point $(500, 300, 3390)$ on the mountain. In what direction should the climber move in order to climb at the greatest rate?

(083)

Q.1 Consider the surface with equation $36x^2 + 9y^2 + 4z^2 = 36$.

i) If $P_0(x_0, y_0, z_0)$ is a point on the given surface, show that the equation of the tangent plane to the surface at P_0 is

$$x_0x + \frac{y_0y}{4} + \frac{z_0z}{9} = 1$$

ii) Identify and sketch the surface.

(082)

Q.5 i) Find the directional derivative of the function $f(x,y) = \ln(x^2+y^2)$ at the point $(1,2)$ in the direction of $\vec{v} = \langle -1, 2\rangle$

ii) Find the maximum rate of change of f at the point $(1,2)$.

(081)

Q.5: Suppose over certain region of space the electrical potential V is given by $V(x,y,x) = 3xy^2 - y^2 + xyz$.

(a) Compute the rate of change of the petential at $A(1,1,-1)$ in the direction of $\vec{u} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$.

(b) In which direction does V changes most rapidly?

(c) What is the maximum rate of change at A?

(073)

5. (a) Find the directional derivative of $f(x,y,z) = x^2 + y^2 + z^2$ at $P(2,1,3)$ in the direction of the origin. (4 marks)

   (b) Find the directions in which the directional derivative of $f(x,y) = x^2 + \sin xy$ at the point $(1,0)$ has the value 1. (4 marks)

OR

(a) Find the directional derivative of $f(x,y) = \sqrt{xy}$ at $P(2,8)$ in the direction of $Q(5,4)$. (4 marks)

(b) Find all points at which the direction of fastest change of the function $f(x,y) = x^2+y^2-2x-4y$ is $\vec{i} + \vec{j}$. (4 marks)

(072)

6. Find the points on the hyperboloid $x^2 - y^2 + 2z^2 = 1$ where the normal line is parallel to the line that joins the points $P(3,-1,0)$ and $Q(5,3,6)$. (7 marks)

7. Find the directions in which the directional derivative of $f(x,y) = x^2 + \sin xy$ at the point $(1,0)$ has the value 1. (7 marks)

(062)

5. [4pts] Find the directional derivative of $f(x,y,z) = (x+y)(y+z)$ at the point $P(5,7,1)$ in the direction of $\vec{v} = \langle -3,0,1 \rangle$.

6. [4pts] Find an equation of the tangent plane and symmetric equations of the normal line to the surface $z + 1 = xe^y \cos z$ at the point $P(1,0,0)$.

(061)

(b) Given that $\nabla f(x_0, y_0) = \vec{i} - 2\,\vec{j}$ and $D_{\vec{u}}\, f(x_0, y_0) = -2$. Find $\vec{u}$. (5 points)

7. Find parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $x^2 + 4y^2 + z^2 = 4$ at the point $(1, -1, 2)$. (5 points)

Old Exam 14.7 :

(091)

b) Find the equation of the plane $\Re_1$ passing through the points $(0,0,0)$, $(1,2,3)$, $(-2,3,3)$, and the distance between the plane $\Re_1$ and the plane $\Re_2$ whose equation is given by

$$-3x - 9y + 7z = 4$$

(083)

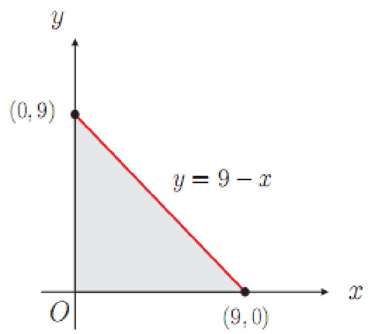
Q.2 Consider the function $f(x,y) = \sqrt{3} + 4xy - x^4 - y^4$.

i) Find all critical points of f.

ii) Find the relative (local) maximum and minimum values and saddle points of f.

(082)

Q.6 Find the absolute maximum and minimum values of $f(x,y) = 2 + 2x + 2y - x^2 - y^2$ on the closed triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$.



(081)

Q.6: Find the maximum and minimum values of $f(x,y) = xy - x^3y^2$ over the region $R = \{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}$.

(073)

6. Find the local maximum and minimum values and saddle points of the function $f(x,y) = x^2ye^{-x^2-y^2}$

(6 marks)

7. Find the extreme value of the function $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$. (6 marks)

OR

Find the shortest distance from the point $(2,1,-1)$ to the plane $x + y - z = 1$. (6 marks)

(063)

6. Find the maximum and minimum of $f(x,y) = x^3 + 3xy - y^3$ in the region bounded by $x = 3$, $y = 3$, $x = 0$, $y = 0$.

(062)

7. [8pts] Find the local maximum and minimum values and the saddle point(s) of the function

$$f(x,y) = \frac{x^3}{3} + \frac{4y^3}{3} - x^2 - 3x - 4y - 3.$$

(061)

8. Locate all relative extrema and saddle points of $f(x,y) = 4xy - x^4 - y^4$.

(10 points)

Old Exam 14.8 :

(092)

13. The plane $x+2y+4z=4$ intersects the cylinder $x^2+y^2=5$ in an ellipse. If P is the highest point on the ellipse and Q is the lowest point on the ellipse, then $\frac{1}{2}(P+Q)$ is:

(a) $(0,0,1)$

(b) $(-1,-2,4)$

(c) $(1,2,4)$

(d) $(0,0,4)$

(e) $(0,0,0)$

(091)

(13) At the point $(0, \pi/4)$, the rate of change of the function

$$f(x,y) = e^{xy}\cos(y)$$

is maximized in the direction of

(A) $\langle 2,1\rangle$

(B) $\langle 1,0\rangle$

(C) $\langle 4,\pi\rangle$

(D) $\langle 0,1\rangle$

(E) $\langle \pi,-4\rangle$

(14) The maximum of

$$f(x, y) = xy$$

subject to the constraint

$$(x+1)^2 + y^2 = 1$$

is equal to

(A) 0

(B) $\sqrt{3}$

(C) $\frac{3\sqrt{3}}{4}$

(D) $\frac{3}{4}$

(E) $\frac{3\sqrt{3}}{8}$

(083)

13. Let P and Q be two points on the sphere $x^2 + y^2 + z^2 = 4a^2$ that are closest to and farthest from $M(3, 1, -1)$, respectively. Then P and Q are given by

(A) $P\left(\frac{6a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}\right)$ and $Q\left(-\frac{6a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}\right)$

(B) $P\left(\frac{6a}{11}, \frac{2a}{11}, -\frac{2a}{11}\right)$ and $Q\left(-\frac{6a}{11}, -\frac{2a}{11}, \frac{2a}{11}\right)$

(C) $P\left(-\frac{6a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}\right)$ and $Q\left(\frac{6a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}\right)$

(D) $P\left(-\frac{6a}{11}, -\frac{2a}{11}, -\frac{2a}{11}\right)$ and $Q\left(\frac{6a}{11}, \frac{2a}{11}, \frac{2a}{11}\right)$

(E) $P\left(\frac{2a}{\sqrt{11}}, \frac{2a}{\sqrt{11}}, -\frac{6a}{\sqrt{11}}\right)$ and $Q\left(-\frac{2a}{\sqrt{11}}, -\frac{2a}{\sqrt{11}}, \frac{6a}{\sqrt{11}}\right)$

(082)

13. The minimum value of the function $f(x, y, z) = 2x + 6y + 10z$ subject to the constraint $x^2 + y^2 + z^2 = 35$ is

(A) -70

(B) -60

(C) 0

(D) 12

(E) -80

(081)

16. The maximum value of $f(x, y, z) = x + 2y - 3z$ subject to the constraint $z = 4x^2 + y^2$ is equal to (Hint: Use Lagrange Multipliers)

(a) $\frac{17}{48}$

(b) 0

(c) $\frac{7}{8}$

(d) 5

(e) -2

(073)

Q.7: Find the point on the plane $x - y + z = 4$ that is closest to the point $(1, 2, 3)$. (do not use Lagrange multiplier) **(10 pts)**

(072)

5. A cardboard box without a lid is to have a volume of 32000 cm^3. Find the dimensions that minimize the amount of cardboard used. (10 marks)

6. Use Lagrange multipliers to find the maximum and minimum volume of a rectangular box whose surface area is 1500 cm^2 and whose total edge length is 200 cm. (10 marks)

(071)

9. If Lagrange multipliers are used to find the maximum M of $f(x, y) = x^2 + y^2$ subject to the constraint $g(x, y) = x^4 + y^4 = 1$, then M is equal to

(a) $3\sqrt{2}$

(b) $2\sqrt{3}$

(c) $2\sqrt{2}$

(d) $\sqrt{3}$

(e) $\sqrt{2}$

Answer Key :

Question	Answer
13 (092)	A
13 (091)	E
14 (091)	C
13 (083)	A
13 (082)	A
16 (081)	A
9 (071)	--

Review Chapter 14

Review Chapter 14 :

(092)

6. The limit $\lim_{(x,y)\to(0,0)} \left(x^2+y^2\right)\ln\left(x^2+y^2\right)$ is equal to:

(a) 0

(b) 1

(c) ∞

(d) -1

(e) 2

7. If $w = x^2\cos(xy)$ then $\frac{\partial w}{\partial x}\left(\frac{1}{2},\pi\right) + 2\pi\frac{\partial w}{\partial y}\left(\frac{1}{2},\pi\right)$ is equal to:

(a) $-\frac{\pi}{2}$

(b) 0

(c) $\frac{\pi}{2}$

(d) $-\frac{1}{2}$

(e) $\frac{1}{2}$

8. Let $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ and let $L(x,y)$ be the linearization of f at $(2,1)$. Using $L(x,y)$, the value of $f(2.1, 1.2)$ is approximately equal to:

(a) 2.467

(b) 4.27

(c) 2.71

(d) 1.90

(e) 3.12

9. For the function z defined implicitly by the equation $xyz = \sin(e^{xyz} + z)$, the value of $\frac{\partial z}{\partial y}(0,-1)$ is:

(a) 0

(b) 1

(c) -1

(d) $\frac{1}{2}$

(e) $-\frac{1}{2}$

10. The points on the hyperboloid $x^2 - y^2 + 2z^2 = 1$ where the normal line is parallel to the line passing through the points $(3, -1, 0)$ and (5,3,6) are:

(a) $\left(\pm\sqrt{\frac{2}{3}}, \mp\frac{2\sqrt{2}}{\sqrt{3}}, \pm\sqrt{\frac{3}{2}}\right)$

(b) $\left(\pm\frac{3}{2}, \mp\frac{8}{3}, \pm\frac{3}{2}\right)$

(c) $\left(\pm\frac{1}{\sqrt{3}}, \mp\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{2}}\right)$

(d) $\left(1, -2, \frac{3}{2}\right)$

(e) $\left(\pm\sqrt{\frac{3}{2}}, \mp\sqrt{6}, \pm\frac{3\sqrt{3}}{2\sqrt{2}}\right)$

11. The point on the plane $x - y + z = 4$ that is closest to the point $(1, 2, 3)$ is:

(a) $\left(\frac{5}{3}, \frac{4}{3}, \frac{11}{3}\right)$

(b) $(5, 2, 1)$

(c) $\left(\frac{7}{2}, 2, \frac{5}{2}\right)$

(d) $(1, 2, 3)$

(e) $(0, 0, 0)$

12. The function $f(x, y) = 7x^2y + 10xy^2$ has:

(a) 1 critical point

(b) 2 critical points

(c) 3 critical points

(d) 4 critical points

(e) no critical point

(091)

(5) Let

$$f(x, y) = \cos(xy).$$

Suppose x and y are functions of s and t with

$$x(-1,1) = \sqrt{\pi}, \qquad y(-1,1) = \frac{\sqrt{\pi}}{3},$$

$$\frac{\partial x}{\partial s}(-1,1) = 6\sqrt{\pi}, \quad \frac{\partial x}{\partial t}(-1,1) = -2\sqrt{\pi},$$

$$\frac{\partial y}{\partial s}(-1,1) = 2\sqrt{\pi}, \quad \frac{\partial y}{\partial t}(-1,1) = 5\sqrt{\pi}.$$

Then $\frac{\partial f}{\partial s}(-1,1)$ is equal to

(A) π

(B) $-\sqrt{3}\pi$

(C) $\frac{-3\pi}{2}$

(D) $-2\sqrt{3}\pi$

(E) $\frac{\pi}{3}$

(6) Given that the function

$$f(x, y) = (x - 1)^2 + (y - 1)^2$$

does not have any critical points in the interior of the rectangular domain

$$D = \{(x, y) : 0 \le x \le 1,\ 0 \le y \le 2\},$$

the absolute maximum value of f on D is

(A) 1
(B) 4
(C) $\frac{1}{2}$
(D) 2
(E) 3

(9) If (a, b) is a critical point of a function f, and if

$$f_{xx}(a, b) = -2 \quad \text{and} \quad f_{yy}(a, b) = 3,$$

then what can one say about (a, b)?

(A) f has a local minimum at (a, b).

(B) Nothing can be concluded from the given information.

(C) f has a saddle point at (a, b).

(D) f has a local maximum at (a, b).

(E) $f_{xxyy}(a, b) = -6$

(10) The equation of the tangent plane to the surface given by the equation

$$x \cos(z) - y^2 \sin(xz) = 2$$

at the point $P(2, 1, 0)$ is

(A) $2x + 3y - 2z = 5$

(B) $x - 2z = 2$

(C) $x - 2y = 2$

(D) $x + y - 2z = 3$

(E) $x - y + 2z = 1$

(12) In an experiment, the temperature of a sample (in degrees Celsius) is given by the function

$$T(x, y, z) = 2y^2 + ze^{-x} + 16,$$

where x, y and z are variables. Using the linear approximation of the function T at the point $(0, 1, 2)$, then $T(0.2, 0.9, 2.3)$ is approximately equal to

(A) 19.5

(B) 19.6

(C) 20.1

(D) 20.2

(E) 19.9

(083)

7. If

$$f(x, y) = \begin{cases} \dfrac{x^2y^3}{2x^2 + y^2}, & (x, y) \neq (0, 0) \\ 1, & (x, y) = (0, 0), \end{cases}$$

then

(A) f is not continuous at $(0, 0)$

(B) $\lim\limits_{(x,y)\to(0,0)} f(x, y) = 1$

(C) $f_x(0, 0) = 1$

(D) f is continuous everywhere

(E) $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ does not exist

8. Let $f(x,t) = \tan^{-1}(x\sqrt{t})$. The value of $f_{xt}(2,1)$ is

(A) $-\dfrac{3}{50}$

(B) 0

(C) $\dfrac{3}{50}$

(D) $\dfrac{1}{10}$

(E) $-\dfrac{1}{10}$

9. If $z = e^y \sin^{-1} x,\ x = \cos(u+v),\ y = u \ln v$, then $\dfrac{\partial z}{\partial v} - \dfrac{\partial z}{\partial u}$ is equal to

(A) $\dfrac{z}{v}(u - v\ln v)$

(B) $\dfrac{y}{v}(u - v\ln v)$

(C) $\dfrac{x}{v}(z - v\ln v)$

(D) $\dfrac{z}{u}(v - u\ln v)$

(E) $\dfrac{z}{x}(u - y\ln v)$

10. Using linear approximation of the function $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ at $(3,2,6)$, the value of $\sqrt{(3.02)^2+(1.97)^2+(5.93)^2}$ is approximately equal to

(A) 6.94

(B) 6.96

(C) 7.04

(D) 7.05

(E) 7.06

11. The points on the hyperboloid $x^2 - y^2 + 2z^2 = 2$ at which the normal line is parallel to the line that contains the points $(3, -1, 0)$ and $(4, 0, -2)$ are

(A) $(1, -1, -1)$ and $(-1, 1, 1)$

(B) $(\sqrt{2}, 0, -1)$ and $(-\sqrt{2}, 0, 1)$

(C) $(-1, -1, 1)$ and $(1, 1, -1)$

(D) $(1, -1, 1)$ and $(-1, 1, -1)$

(E) $(\sqrt{2}, 0, 1)$ and $(-\sqrt{2}, 0, -1)$

12. Let $f(x, y) = 2x^2 - 4x + y^2 - 2y + 1$ and D be the closed triangular region bounded by $x = 0$, $y = 2$ and $y = 2x$ in the first quadrant. The absolute minimum value of f is equal to

(A) $-\frac{5}{3}$

(B) -2

(C) -1

(D) 0

(E) $-\frac{4}{3}$

(082)

7. Let $L = \lim_{(x,y)\to(0,0)} \dfrac{\sin(\sqrt{x^2 + y^2})}{\sqrt{2x^2 + 2y^2}}$. Then

(A) $L = \dfrac{\sqrt{2}}{2}$

(B) $L = \sqrt{2}$

(C) $L = 0$

(D) L does not exist

(E) $L = 1$

8. Let $f(x,t) = x^2 e^{-t/2}$. The partial derivative $f_{txx}(x,t)$ is

(A) $-e^{-t/2}$

(B) $e^{-t/2}$

(C) $xe^{-t/2}$

(D) $-xe^{-t/2}$

(E) e^{-t}

9. Using the linear approximation of the function $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ at $(2,1)$, the value of $f(1.97, 1.06)$ is approximately equal to

(A) 2.88

(B) 2.98

(C) 3.08

(D) 3.16

(E) 3.06

10. Let $z = \tan^{-1}(u/v)$, where $u(x,y) = 2x + y$ and $v(x,y) = 3x - y$. Then $\partial z/\partial y$ at the point $(x,y) = (1,1)$ is equal to

(A) $\frac{5}{13}$

(B) 0

(C) $\frac{3}{13}$

(D) $\frac{1}{13}$

(E) $\frac{7}{13}$

11. The points on the surface $x^2 + 2y^2 + 3z^2 = 12$ at which the tangent plane is perpendicular to the line with parametric equations $x = 1 + 2t$, $y = 3 + 8t$, $z = 2 - 6t$ are

(A) $(1, 2, -1)$ and $(-1, -2, 1)$

(B) $(-1, 2, 1)$ and $(1, -2, -1)$

(C) $(-1, -2, -1)$ and $(1, 2, 1)$

(D) $(2, -2, 0)$ and $(-2, 2, 0)$

(E) $(2, 2, 0)$ and $(-2, -2, 0)$

12. If $f(x,y) = -y^3 + 4xy - 2x^2 + 1$, then f has

(A) a local maximum at $(\frac{4}{3}, \frac{4}{3})$ and a saddle point at $(0,0)$

(B) a local maximum at $(0,0)$ and a local minimum at $(\frac{4}{3}, \frac{4}{3})$

(C) a local maximum at $(0,0)$ and a saddle point at $(\frac{4}{3}, \frac{4}{3})$

(D) a local maximum at $(0,0)$ and $(\frac{4}{3}, \frac{4}{3})$

(E) a local minimum at $(\frac{4}{3}, \frac{4}{3})$ and a saddle point at $(0,0)$

(081)

9. Let $f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & (x,y) \neq (0,0) \\ 3 & (x,y) = (0,0) \end{cases}$

and $L = \lim\limits_{(x,y)\to(0,0)} f(x,y)$. Then

(a) L does not exist

(b) $L = 3$

(c) $L = 0$ and $f(x,y)$ is not continuous at $(0,0)$.

(d) $L = 1$ and $f(x,y)$ is not continuous at $(0,0)$.

(e) $L = 3$ and $f(x,y)$ is not continuous at $(0,0)$.

10. If $u = e^{ax+by+cz}$, where $a^2 + b^2 + c^2 = 6$, then $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}$ is equal to

(a) $6u$

(b) u

(c) $\dfrac{6}{u}$

(d) $6u^2$

(e) u^2

11. Let $z = f(x, y), x = r\cos\theta, y = r\sin\theta$. Then $\left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2$ is equal to

(a) $\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2$

(b) $\left(\dfrac{\partial z}{\partial x}\right)^2 - \left(\dfrac{\partial z}{\partial y}\right)^2$

(c) $\left(\dfrac{\partial z}{\partial x}\right)^2 + \dfrac{1}{x^2+y^2}\left(\dfrac{\partial z}{\partial y}\right)^2$

(d) $\left(\dfrac{\partial z}{\partial x}\right)^2 \cdot \left(\dfrac{\partial z}{\partial y}\right)^2$

(e) $(x^2+y^2)\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2$

12. Using the linear approximation of $f(x, y) = \sqrt{x^2 + y^2}$ at the point $(3, 4)$, the value of $\sqrt{(2.9)^2 + (4.1)^2}$ is approximately equal to

(a) $\dfrac{251}{50}$

(b) $\dfrac{257}{50}$

(c) $\dfrac{1}{50}$

(d) $\dfrac{6}{5}$

(e) $\dfrac{3}{25}$

13. The directional derivative of $f(x, y) = x^2 + \sin(xy)$ at the point $(1, 0)$ is equal to 1 in the direction of the unit vectors

(a) $\langle 0, 1\rangle$ and $\langle \frac{4}{5}, -\frac{3}{5}\rangle$

(b) $\langle 1, 0\rangle$ and $\langle \frac{4}{5}, -\frac{3}{5}\rangle$

(c) $\langle -1, 0\rangle$ and $\langle -\frac{4}{5}, \frac{3}{5}\rangle$

(d) $\langle 1, 0\rangle$ and $\langle -\frac{4}{5}, -\frac{3}{5}\rangle$

(e) $\langle 0, 1\rangle$ and $\langle -\frac{4}{5}, -\frac{3}{5}\rangle$

14. The function $f(x, y) = x^4 + y^4 - 4xy + \sqrt{5}$ has

(a) Local minimum at $(1, 1), (-1, -1)$ and saddle point at $(0, 0)$

(b) Local minimum at $(1, 1), (-1, -1), (1, -1), (-1, 1)$ and saddle point at $(0, 0)$

(c) Local maximum at $(1, 1), (-1, -1)$ and saddle point at $(0, 0)$

(d) Local minimum at $(-1, -1)$, local maximum at $(1, 1)$ and saddle point at $(0, 0)$

(e) Local minimum at $(1, 1)$, local maximum at $(-1, -1)$ and saddle point at $(0, 0)$

15. Determine the nature of the critical points $(1,2), (-2,3)$, and $(-1,-1)$ of the function $g(x,y)$ if

$$\begin{array}{ccc} g_{xx}(1,2) = 2 & g_{yy}(1,2) = 3 & g_{xy}(1,2) = 2 \\ g_{xx}(-2,3) = -4 & g_{yy}(-2,3) = 5 & g_{xy}(-2,3) = 4 \\ g_{xx}(-1,-1) = -3 & g_{yy}(-1,-1) = -4 & g_{xy}(-1,-1) = 3 \end{array}$$

(a) Local minimum at $(-1,-1)$, Local minimum at $(1,2)$, Saddle point at $(-2,3)$.

(b) Local maximum at $(1,2)$, Local minimum at $(-1,-1)$, Saddle point at $(-2,3)$.

(c) Local maximum at $(-2,3), (-1,-1)$, Local minimum at $(1,2)$.

(d) Local minimum at $(1,2)$, Saddle point at $(-2,3)$.

(e) Local minimum at $(-1,-1)$, Local maximum at $(1,2)$, $(-2,3)$.

Answer Key :

Question	Answer
6 (092)	A
7 (092)	A
8 (092)	A
9 (092)	A
10 (092)	A
11 (092)	A
12 (092)	A
5 (091)	D
6 (091)	D
9 (091)	C
10 (091)	B
12 (091)	A
7 (083)	A
8 (083)	A
9 (083)	A
10 (083)	A
11 (083)	A
12 (083)	A
7 (082)	A
8 (082)	A
9 (082)	A
10 (082)	A
11 (082)	A
12 (082)	A
9 (081)	A
10 (081)	A
11 (081)	A
12 (081)	A
13 (081)	A
14 (081)	A
15 (081)	A

Review Chapter 15

Review Chapter 15：

(092)

14. An approximate value of the integral $\iint\limits_R (2\cos^2 y - \sin^2 x)\, dA$ where $R = \left[0, \frac{\pi}{2}\right] \times \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$, using $m = 3, n = 2$ and with upper left corner points is equal to:

(a) $\frac{17\pi^2}{72}$

(b) $-\frac{17\pi}{72}$

(c) $\frac{19\pi^2}{72}$

(d) $-\frac{20\pi}{3}$

(e) 6

15. The value of $\iint\limits_R \frac{1 + x^2 y}{1 + y^2} dA$, where $R = [0, 1] \times [0, 1]$ is equal to:

(a) $\frac{\pi}{4} + \frac{\ln 2}{6}$

(b) $\frac{\pi}{2} + \frac{\ln 2}{3}$

(c) $\frac{\pi}{4} + \frac{\ln 2}{3}$

(d) $\frac{\pi}{2} + \frac{\ln 2}{6}$

(e) $\frac{\pi}{3} + \frac{\ln 2}{3}$

16. The value of $\int_0^1 \int_{3y}^3 e^{x^2}\, dx dy$ is :

(a) $\dfrac{e^9 - 1}{6}$

(b) $\dfrac{e^9}{6}$

(c) $\dfrac{2(e^9 - 1)}{6}$

(d) $e^9 - e^{\frac{9}{4}}$

(e) $e^9 - e^{\frac{9}{4}} + 2$

17. The value of the double integral $\int\limits_0^4 \int\limits_0^{\sqrt{16-x^2}} e^{-\left(x^2+y^2\right)} dy dx$ is equal to:

(a) $\dfrac{\pi\left(1 - e^{-16}\right)}{4}$

(b) $\dfrac{\pi\left(1 + e^{-16}\right)}{4}$

(c) $\dfrac{\pi\left(1 - e^{-16}\right)}{2}$

(d) $\dfrac{\pi\left(1 + e^{16}\right)}{4}$

(e) $\dfrac{\pi\left(1 - e^{16}\right)}{4}$

18. The value of the integral $\iiint\limits_E (x+2y)\,dV$, where E is bounded by the the parabolic cylinder $y = x^2$ and the planes $y = x, z = 0$ and $z = x$ is:

(a) $\dfrac{2}{15}$

(b) $\dfrac{4}{15}$

(c) $\dfrac{13}{15}$

(d) $\dfrac{1}{15}$

(e) $\dfrac{7}{15}$

19. The value of $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)^{\frac{3}{2}}dzdydx$ is:

(a) $\dfrac{64\pi}{15}$

(b) $\dfrac{64\pi}{5}$

(c) $\dfrac{128\pi}{5}$

(d) $\dfrac{128\pi}{15}$

(e) $\dfrac{64}{15}$

20. The value of the integral $\iiint\limits_E \frac{e^{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}} dV$, where $E = \left\{(x, y, x) \,|x^2 + y^2 + z^2 \leq 1\right\}$, is:

(a) $2\pi\,(e-1)$

(b) $2\pi e$

(c) $\pi\,(e-1)$

(d) πe

(e) $4\pi\,(e-1)$

(091)

(8) Consider the sphere

$$x^2 + (y-3)^2 + z^2 = 25$$

and the cylinder

$$x^2 + y^2 = 4.$$

The volume of the solid region inside both the sphere and the cylinder is given in cylindrical coordinates by

(A) $\int_0^{2\pi} \int_0^2 \int_{-\sqrt{16r\sin\theta}}^{\sqrt{16-r^2}} r dz dr d\theta$

(B) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 \int_{-\sqrt{16-r^2+6r\sin\theta}}^{\sqrt{16-r^2+6r\sin\theta}} r dz dr d\theta$

(C) $\int_0^{2\pi} \int_0^2 \int_{-\sqrt{16-r^2+6r\sin\theta}}^{\sqrt{16-r^2+6r\sin\theta}} r dz dr d\theta$

(D) $\int_0^{2\pi} \int_0^2 \int_{-\sqrt{16-r^2+6r\sin\theta}}^{\sqrt{16-r^2+6r\sin\theta}} dr dz d\theta$

(E) $\int_0^1 \int_0^{2\pi} \int_0^{\sqrt{16-r^2+6r\sin\theta}} r dz dr d\theta$

(11) The integral that gives the volume of the solid region inside the sphere centered at the origin with radius 19 and between the cones $\phi = \frac{\pi}{4}$ and $\phi = \frac{\pi}{6}$ is

(A) $\int_0^{2\pi} \int_{\pi/4}^{\pi/6} \int_0^{19/2} \rho \sin\phi \, d\rho d\phi d\theta$

(B) $\int_0^{2\pi} \int_{\pi/6}^{\pi/4} \int_0^{19} \sin^2\phi \, d\rho d\phi d\theta$

(C) $\int_0^{2\pi} \int_{\pi/6}^{\pi/4} \int_0^{19} \rho \cos\phi \, d\rho d\phi d\theta$

(D) $\int_0^{2\pi} \int_{\pi/6}^{\pi/4} \int_0^{19} \rho^2 \sin\phi \, d\rho d\phi d\theta$

(E) $\int_0^{\pi} \int_{\pi/6}^{\pi/4} \int_{-19}^{19} \rho^2 \sin\phi \, d\rho d\phi d\theta$

(15) Let E be the solid region that lies under the plane $z = 1 + x + y$ and above the region of the xy–plane in the first quadrant bounded by the curves $x = y^2$, $y = 0$, and $x = 1$. Then

$$\iiint_E 2xy dV$$

is equal to

(A) $\frac{65}{28}$

(B) $\frac{65}{84}$

(C) $\frac{65}{42}$

(D) $\frac{65}{12}$

(E) $\frac{65}{14}$

(16) The value of the double integral

$$\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3) dy dx$$

is equal to

(A) $\frac{\sin 1}{3}$

(B) $\frac{\sin 1}{4}$

(C) $\frac{\cos 1}{3}$

(D) $\frac{4\cos 1}{3}$

(E) $\frac{\sin 1 \cos 1}{3}$

(17) Let D be the disc centered at the point $(1,0)$ with radius 1. Using polar coordinates, the value of the double integral

$$\iint\limits_D \sqrt{x^2+y^2}dA$$

is equal to

(A) $\frac{4}{3}\int_{-\pi/2}^{\pi/2} \cos^2\theta\, d\theta$

(B) $\frac{3}{8}\int_{0}^{\pi/2} \cos^3\theta\, d\theta$

(C) 0

(D) $\frac{16}{3}\int_{0}^{\pi/2} \cos^3\theta\, d\theta$

(E) $\frac{16}{3}\int_{0}^{1} r^3 dr$

(083)

14. The volume of the solid bounded by the surface $\rho = a$ is given by

(A) $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho^2 \sin\phi\; d\rho d\phi d\theta$

(B) $\displaystyle\int_0^{2\pi}\int_0^{2\pi}\int_0^{a} \rho^2 \sin\phi\; d\rho d\phi d\theta$

(C) $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{a} d\rho d\phi d\theta$

(D) $\displaystyle\int_0^{\pi}\int_0^{\pi}\int_{-a}^{a} d\rho d\phi d\theta$

(E) $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_{-a}^{a} \rho^2 \sin\phi\; d\rho d\phi d\theta$

15. The volume of the solid bounded by $x+y+z=3$ and the planes $x=0$, $x=a$, $y=0$, $y=b$ and $z=0$ is

(A) $\frac{ab}{2}(6-a-b)$

(B) $\frac{ab}{2}(a^2-b^2)$

(C) $ab(a^2+b^2)$

(D) a^3+b^3

(E) $ab(a^3+b^3)$

16. If $R=\{(x,y)|\ 0\le x\le a,\ \ 0\le y\le b\}$,

then $\displaystyle\iint_R (xy^4+yx^4)\ dA$ is equal to

(A) $\frac{a^2b^2}{10}(a^3+b^3)$

(B) $\frac{a^3b^3}{10}(a^2+b^2)$

(C) $\frac{a^2b^2}{10}(a^3-b^3)$

(D) $\frac{a^2b^2}{10}(b^3-a^3)$

(E) $\frac{a^2b^2}{10}$

17. The volume of the solid bounded by $z=a$ and $z=a\sqrt{x^2+y^2}$ $(a>0)$ is

(A) $\frac{\pi a}{3}$

(B) $\frac{4\pi a}{3}$

(C) $\frac{\pi a^2}{3}$

(D) $\frac{\pi a^3}{3}$

(E) $\frac{2\pi a^3}{3}$

18. Let E be the solid outside the cylinder $x^2 + y^2 = 1,\ 0 \leq z \leq 1$ and inside the hemisphere $x^2 + y^2 + z^2 = 4,\ z \geq 0$. The volume of E is equal to

(A) $2\sqrt{3}\pi$

(B) 2π

(C) $\sqrt{3}\pi$

(D) 4π

(E) π

19. Let E be the solid bounded by $z = 0$, $z = x^2 + y^2$, $y = x^2$ and $x = y^2$. Then the volume of E is equal to

(A) $\dfrac{6}{35}$

(B) $\dfrac{3}{14}$

(C) $\dfrac{5}{4}$

(D) $\dfrac{7}{35}$

(E) $\dfrac{1}{5}$

20. Let D be the region bounded by the line $y = x-1$ and the parabola $y^2 = 2x+6$

The value of the double integral $\displaystyle\iint_D \sqrt{3}xy\ dA$ is equal to

(A) $36\sqrt{3}$

(B) $24\sqrt{3}$

(C) $12\sqrt{3}$

(D) $\sqrt{3}$

(E) $2\sqrt{3}$

(082)

14. Using a Riemann sum with $m = n = 2$ and lower left corners as the sample points, the approximate value of the double integral

$$\iint_R (x + 2y)\,dA,$$

where $R = [0, 1] \times [0, 1]$ is

(A) $\dfrac{3}{4}$

(B) 1

(C) -2

(D) $-\dfrac{2}{3}$

(E) $\dfrac{3}{2}$

15. The volume of the solid bounded by the surface $z = 2x\sqrt{x^2 + y}$ and the planes $x = 0$, $x = 1$, $y = 0$, $y = 3$ and $z = 0$ is

(A) $\dfrac{4}{15}(31 - 9\sqrt{3})$

(B) $\dfrac{4}{15}(32 - 9\sqrt{3})$

(C) $\dfrac{2}{15}(32 - 9\sqrt{3})$

(D) $\dfrac{2}{15}(31 - 9\sqrt{3})$

(E) $\dfrac{1}{15}(31 - 9\sqrt{3})$

16. Let $R = \{(x, y) |\ 0 \leq x \leq \pi, \quad 0 \leq y \leq \pi/2\}$. Then

$$\iint_R \cos(x + 2y)\, dA$$

is equal to

(A) -2

(B) -4

(C) 0

(D) π

(E) $\pi/2$

17. The value of the iterated integral

$$\int_0^1 \int_{\sin^{-1} y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x}\ dx dy$$

is equal to

(A) $\dfrac{2\sqrt{2} - 1}{3}$

(B) $\dfrac{\pi}{3}$

(C) $\pi - 2\sqrt{2}$

(D) $\dfrac{2\sqrt{2}}{3}$

(E) $\dfrac{2}{3}$

18. The value of the double integral

$$\iint_D e^{x/y}\, dA,$$

where $D = \{(x, y)|1 \leq y \leq 2,\ y \leq x \leq y^3\}$, is equal to

(A) $\dfrac{e^4 - 4e}{2}$

(B) $\dfrac{e^3 - 3}{2}$

(C) $\dfrac{e^4}{2}$

(D) $\dfrac{1}{4}$

(E) $\dfrac{e^2 - 2e}{4}$

19. The value of the double integral

$$\iint_R \sqrt{4 - x^2 - y^2}\, dA,$$

where $R = \{(x, y)|\ x^2 + y^2 \leq 4,\ x \geq 0\}$, is equal to

(A) $\dfrac{8\pi}{3}$

(B) $\dfrac{8\pi}{5}$

(C) $\dfrac{8\pi}{7}$

(D) π

(E) 2π

20. The value of the double integral

$$\iint\limits_R \tan^{-1}\left(\frac{y}{x}\right)\, dA,$$

where $R = \left\{(x, y) | 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq \sqrt{3}\, x\right\}$, is equal to

(A) $\dfrac{\pi^2}{12}$

(B) $\dfrac{\pi^2}{18}$

(C) $\dfrac{3\pi^2}{8}$

(D) $\dfrac{\pi}{12}$

(E) $\dfrac{3\pi}{16}$

(081)

17. The volume of the solid that lies under the paraboloid $z = 2b^2\, x^2 + a^2 y^2\, (a, b > 0)$ and above the rectangle $[0, a] \times [0, b]$ is

(a) $(ab)^3$

(b) $(a + b)^3$

(c) $a^2 b + ab^2$

(d) $a^3 + b^3$

(e) 1

18. The volume of the solid bounded by the surface $z = x\sqrt{x^2+y}$ and the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, and $z = 0$ is

(a) $\frac{2}{15}\left(2^{\frac{5}{2}} - 2\right)$

(b) $\frac{2}{15}\left(2^{\frac{5}{2}} + 2\right)$

(c) $\frac{2}{15}\left(2^{\frac{5}{2}} - 1\right)$

(d) $\frac{3}{15}\left(2^{\frac{5}{2}} + 2\right)$

(e) $\frac{4}{15}\left(2^{\frac{5}{2}} - 1\right)$

19. The volume of the solid under the surface $z = 2x + y^2$ and above the region in xy-plane bounded by $x = y^2$ and $x = y^3$ is

(a) $\frac{19}{210}$

(b) $\frac{18}{210}$

(c) $\frac{1}{7}$

(d) $\frac{2}{5}$

(e) $\frac{13}{42}$

20. The value of the iterated integral $\int_0^2 \int_{2x}^4 e^{y^2}\, dy\, dx$ is equal to

(a) $\frac{1}{4}\left(e^{16} - 1\right)$

(b) $\frac{1}{2}\left(e^{16} + 1\right)$

(c) $\frac{1}{4}\left(e^{16} + 1\right)$

(d) $\frac{1}{2}\left(e^{16} - 1\right)$

(e) $\frac{1}{8}\left(e^{16} - 2\right)$

21. The value of the iterated integral $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{x^2+y^2}\, dy\, dx$ is equal to

(a) $\frac{\pi}{4}\left(e^4 - 1\right)$

(b) $\frac{\pi}{2}\left(e^2 - 1\right)$

(c) $\frac{\pi}{4}\left(e^4 + 1\right)$

(d) $\frac{\pi e^2}{4}$

(e) $\frac{\pi e^{16}}{4}$

22. If volume of a tetrahedron formed by the plane $ax + y - z = 4$ and the three coordinate planes is $\frac{16}{3}$, then value of a is

(a) -2

(b) 2

(c) -3

(d) 4

(e) 0

23. The volume of the solid enclosed by the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$ is

(a) $\frac{16}{3}$

(b) 8

(c) $\frac{2}{3}$

(d) 4

(e) 1

24. The value of $\iiint\limits_E \sqrt{x^2+y^2+z^2}\,dV$, where E is the solid that lies between the spher
$x^2+y^2+z^2=4$ and $x^2+y^2+z^2=9$ and above the xy-plane, is

(a) $\dfrac{65\pi}{2}$

(b) $\dfrac{65\pi}{4}$

(c) $\dfrac{56\pi}{4}$

(d) $\dfrac{65\pi^2}{4}$

(e) $\dfrac{65\pi}{8}$

25. The triple integral that gives the volume of the solid that lies inside the sphere $x^2+y^2+z^2=2$ and outside the cone $z^2=x^2+y^2$ is

(a) $\displaystyle\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_0^{\sqrt{2}} \rho^2\sin\phi d\rho d\phi d\theta$

(b) $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{2}} \rho^2\sin\phi d\rho d\phi d\theta$

(c) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^{\pi}\int_{-\sqrt{2}}^{\sqrt{2}} \rho^2\sin\phi d\rho d\theta d\phi$

(d) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}}\int_0^{2\pi}\int_0^{\frac{\pi}{4}} \rho^2\sin\phi d\phi d\theta d\rho$

(e) $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\sqrt{2}}\int_{\frac{\pi}{2}}^{\frac{7\pi}{4}} \rho^2\sin\phi d\phi d\rho d\theta$

Answer Key :

Question	Answer
14 (092)	A
15 (092)	A
16 (092)	A
17 (092)	A
18 (092)	A
19 (092)	A
20 (092)	A
8 (091)	C
11 (091)	D
15 (091)	B
16 (091)	A
17 (091)	D
14 (083)	A
15 (083)	A
16 (083)	A
17 (083)	A
18 (083)	A
19 (083)	A
20 (083)	A
14 (082)	A
15 (082)	A
16 (082)	A
17 (082)	A
18 (082)	A
19 (082)	A
20 (082)	A
17 (081)	A
18 (081)	A
19 (081)	A
20 (081)	A
21 (081)	A
22 (081)	A
23 (081)	A
24 (081)	A
25 (081)	A

الخاتمة

سبحان ربك رب العزة عما يصفون ، وسلام على المرسلين ، و الحمد لله رب العالمين .

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سبحانك اللهم و بحمدك ، أشهد أن لا إله إلا أنت ، أستغفرك و أتوب إليك .